Print your name clearly:____

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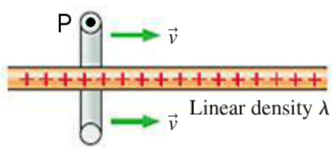
"I agree to abide by the Duke Community Standard and will neither give nor receive aid during this quiz."

Physics 162: Quiz 10 (Final Quiz, Yay!)

Professor Greenside
Wednesday, April 15, 2015

1. As shown in this figure



a circular metal ring of radius R travels with constant speed $v \ll c$ to the right along an infinitely long, straight, non-conducting string that has a constant positive linear charge density $\lambda > 0$. The string is at rest in the laboratory frame and it passes through the center of the ring.

   (a) **(4 points)** As measured by devices sitting still in the laboratory, what is the electric field **E** (strength and direction) and magnetic field **B** (strength and direction) at the point P at the top of the ring?

   (b) **(4 points)** Using the field transformation equations, calculate the fields **E** and **B** at the point P as measured by a scientist in a frame of reference such that the ring is motionless. (This is the "ring frame of reference".)

   (c) **(4 points)** What values do you get for the fields **E** and **B** at point P in the ring frame of reference if you calculate these fields directly from their sources (charges and current) in the ring frame of reference? Do you get the same values as from the field transformation equations?

   (d) **(3 points)** True or false and briefly why: there is a current flowing inside the metal ring.

2. **(10 points)** As a simple model of the long linear fluorescent light bulbs in the ceiling of this room, consider a line segment of length L that emits light in all directions with total power P. A tiny spherical mirror of radius R is placed a distance d from the line segment, on its perpendicular bisector. Write down a mathematical expression for the total force vector **F** acting on the sphere because of radiation pressure from the glowing line segment.

   Note: you do **not** have to evaluate your expression, just show me how to set it up. To get started, divide the glowing line segment into small identical point-like sources of length dx and labeled by some coordinate x....

3. **(14 points)** A key step in Maxwell's theoretical unification of electrical and magnetic phenomena was his addition of a so-called "displacement current" to Ampère's law, giving the so-called Ampère-Maxwell equation:

$$\oint_C \mathbf{B} \bullet d\mathbf{l} = \mu_0 \left[I_{\text{enclosed}} + \epsilon_0 \frac{d}{dt} \int_S \mathbf{E} \bullet d\mathbf{A} \right] \tag{1}$$

The displacement current turned out to be the key insight that allowed Maxwell to show that his equations had solutions corresponding to self-sustaining transverse electromagnetic waves that could propagate through empty space with speed $1/\sqrt{\epsilon_0 \mu_0} \approx 3 \times 10^8$ m/s.

In the context of some specific physical example for which the displacement current is nonzero, explain what each symbol and term means in Eq. (1). You can choose any example you like but you need to draw and describe a figure that explains your example. Examples of symbols to explain in the context of your figure are C, S, $\mathbf{B}$, $d\mathbf{l}$, I_{enclosed}, $\mathbf{E}$, $d\mathbf{A}$, and the two integrals.

4. The electric field of an electromagnetic wave in vacuum is given by $E_x = E_z = 0$ and

$$E_y(x, y, z, t) = (20\,\text{V/m}) \cos\left[\left(6.28 \times 10^8\right) z + \omega t\right], \tag{2}$$

where the coordinates x, y, and z are measured in meters and the time t is measured in seconds.

   (a) **(4 points)** Write down a mathematical expression for the magnetic field $\mathbf{B}(x, y, z, t)$ of this wave, including a numerical value for its magnitude in tesla to the nearest power of ten.

   (b) **(2 points)** What is the direction of propagation of this wave?

   (c) **(4 points)** To the nearest power of ten and in SI units, what are the wavelength λ and frequency f of this wave?

   (d) **(4 points)** Assuming this wave was produced by passing light through a linear polarizer, discuss briefly how the plane of the polarizing filter and its axis would have to be oriented.

5. Four ideal linear polarizers ("ideal" means that there is no loss of light from reflection or absorption as light passes through the polarizer) are stacked one by behind the other such that the polarizing axes of the second, third, and fourth polarizers make angles of θ_1, θ_2 and $\pi/2$ with respect to the axis of the first filter.

   (a) **(6 points)** If unpolarized light with initial intensity I_0 strikes the first polarizer, give an expression for the light intensity I coming out of the final polarizer.

   (b) **(3 points)** What choices of the angles θ_1 and θ_2 for the intermediate polarizers will minimize the output intensity I and what is the minimal value obtained?

   (c) **Optional (3 points)** What choices of angles θ_1 and θ_2 will maximize I and what is the maximum value?

Some equations:

$$\text{For } v \ll c: \qquad \mathbf{E}_B = \mathbf{E}_A + \mathbf{v}_{BA} \times \mathbf{B}_A \quad \text{and} \quad \mathbf{B}_B = \mathbf{B}_A - \frac{1}{c^2}\mathbf{v}_{BA} \times \mathbf{E}_A. \tag{3}$$

$$\mathbf{E} = E_0\,\hat{\mathbf{n}} \cos(\mathbf{k} \bullet \mathbf{x} - \omega t), \quad \hat{\mathbf{n}} \bullet \hat{\mathbf{k}} = 0, \quad k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \quad c = f\lambda = \frac{\omega}{k}, \quad I = I_0 \cos^2(\theta). \tag{4}$$

$$E_0 = cB_0, \qquad \mathbf{S} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_0}, \qquad I = \frac{\text{power}}{\text{area}} = |\mathbf{S}| = \frac{1}{2} c \epsilon_0 E_0^2, \qquad p = \frac{I}{c} \quad \text{or} \quad \frac{2I}{c}. \tag{5}$$