Physics 162: Quiz 8
Professor Greenside
Wednesday, April 1, 2015

This 30-minute quiz is closed book and nothing is allowed on your desk except items to write with. Please write all answers on the blank pages handed out in class. Please also write your name and problem number on each page, order the pages in increasing order of problem number, and staple all of your pages together at the end of the quiz.

Unless stated otherwise, you need to justify your reasoning, e.g., with brief phrases and drawings; a correct answer without some explanation will get zero credit. If your answers are not easily readable and understandable by the grader, you will lose credit and possibly get no credit.

1. **(5 points)** A resistor with resistance $R$, a capacitor with capacitance $C$, and an inductor with inductance $L$ are connected in series with a battery with emf $E$. Derive a differential equation that describes how the charge $Q(t)$ on the positive capacitor plate evolves in time.

2. **(10 points)** A current $I$ flows through solenoid 1 which has a length $w$, radius $R_1$, and $n_1$ coils per meter. As shown in this figure,

![Diagram of two solenoids](image)

solenoid 1 is placed inside and coaxial with a second solenoid, solenoid 2, which also has length $w$, a larger radius $R_2 > R_1$ and winding density $n_2$. The same current $I$ that flows through solenoid 1 also flows through solenoid 2 but in an opposite direction according to the arrows in the figure. Both solenoids are sufficiently long (length much greater than the radius) that the magnetic fields they generate can be considered uniform.

Derive and give an expression for the inductance $L = |\phi_m|/I$ of this double solenoid in terms of $w$, $R_1$, $R_2$, $n_1$, and $n_2$.

Note: keep in mind that if you have two coils of wire adjacent to each other, the flux of an external magnetic field through the two coils is twice the flux through just one of the coils.
3. An L-shaped conductor slides with constant speed $v$ diagonally down to the right across a stationary L-shaped conductor in the presence of a uniform magnetic field of strength $B$ that comes out of the page as shown in this figure:

The conductors both have a resistance per unit length of $r$ (units of ohms per meter).

(a) **(6 points)** Deduce and give the direction of the induced current in two different ways: first by thinking about this problem as an example of motional emf, and second by using Lenz’s law. (Make sure to briefly explain your reasoning.)

(b) **(10 points)** In terms of $v$, $B$, and $r$, find an expression for the force $F(t)$ (direction and magnitude) needed to keep the L-shaped conductor moving with its constant velocity $v$. A hint is to use conservation of energy to equate “the mechanical power $F \cdot v$ needed to keep the L-shaped bar moving” to “the electrical power dissipated by the resistance of this circuit”.

4. **(10 points)** Three identical lightbulbs A, B, and C, and two identical inductors $L_1$ and $L_2$ are connected to a battery and to a switch S as shown here:

After the switch has been opened for a long time, the switch is closed, and a very short time after that let $A_1$, $B_1$, and $C_1$ be the brightnesses of the three bulbs. A long time after the switch has been closed, let $A_2$, $B_2$, and $C_2$ be the brightnesses of the bulbs. Finally, after the switch has been closed for a long time, it is opened and a very short time after that let $A_3$, $B_3$, $C_3$ denote the brightnesses of the three bulbs. List the nine numbers $A_1$, $B_1$, $C_1$, $A_2$, $B_2$, $C_2$, $A_3$, $B_3$, and $C_3$ in **decreasing** order.
5. (5 points) As shown in the following figure, a circular coil of 15 turns and of radius 1 cm rotates at a constant angular velocity $\omega = 300 \text{ rad/s}$ about the $z$-axis in a uniform magnetic field $\mathbf{B} = -B_0 \mathbf{\hat{x}}$ where $B_0 = 0.5 \text{ T}$:

Assume at time $t = 0$ that the normal $\mathbf{\hat{n}}$ to the coil plane is along the $\mathbf{\hat{y}}$-direction and that the self-inductance of the coil can be neglected. If the coil resistance is 9 $\Omega$, which of the following expressions gives the induced current in milliamperes, as a function of time $t$?

(a) $225\pi \sin(\omega t)$  (b) $250\pi \sin(\omega t)$  (c) $0.08\pi \cos(\omega t)$  
(d) $1.7\pi \cos(\omega t)$  (e) $25\pi \cos(\omega t)$

Some equations:

\[ F = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad d\mathbf{F} = I \, d\mathbf{l} \times \mathbf{B}, \quad \mathbf{\tau} = \mu \times \mathbf{B}, \quad \mu = NIA. \]

\[ \Delta V_R = IR, \quad \Delta V_C = \frac{Q}{C}, \quad \Delta V_L = -L \frac{dI}{dt}, \quad P = IV = I^2 R = \frac{V^2}{R}. \]

\[ B = \mu_0 n I, \quad \Phi_m = \int_S \mathbf{B} \cdot d\mathbf{A}, \quad \mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = \left| \frac{d\Phi_m}{dt} \right|. \]