

Print your name clearly: _____

Signature: _____

"I agree to abide by the Duke Community Standard and will neither give nor receive aid during this quiz."

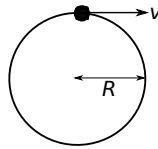
Physics 162: Quiz 7

Professor Greenside
Wednesday, March 25, 2015

This 30-minute quiz is closed book and nothing is allowed on your desk except items to write with. Please write all answers on the blank pages handed out in class. Please also write your name and problem number on each page, order the pages in increasing order of problem number, and staple all of your pages together at the end of the quiz.

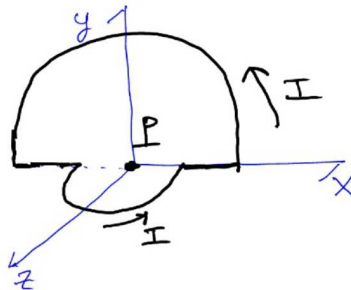
Unless stated otherwise, you need to justify your reasoning, e.g., with brief phrases and drawings; a correct answer without some explanation will get zero credit. If your answers are not easily readable and understandable by the grader, you will lose credit and possibly get no credit.

1. As shown in this figure:



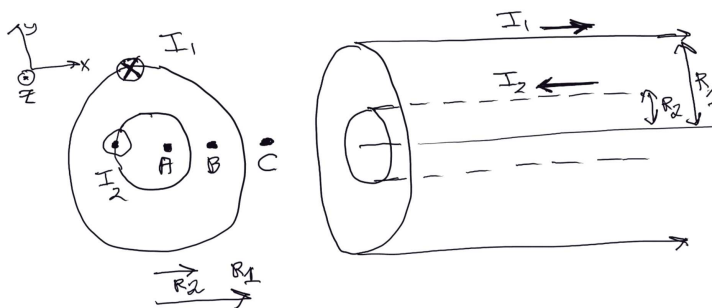
a point particle of mass m and negative charge $q < 0$ moves clockwise with constant speed v in endless circular motion of radius R in the plane of the paper, in a region of space that is filled with a uniform magnetic field of magnitude B .

- (a) **(3 points)** What is the direction of the uniform magnetic field $\mathbf{B}$ causing this circular motion?
- (b) **(6 points)** Show that, surprisingly, the time T needed to complete one circular orbit (the period of the motion) is independent of the circle's radius R , and is independent of the particle's speed v .
2. A constant current I flows as indicated through the following bent rigid wire loop:



that consists of a larger semicircle of radius R_1 that lies in the xy -plane that is connected by short straight wire segments to a smaller semicircle of radius $R_2 < R_1$ that lies in the xz -plane.

- (a) **(8 points)** In terms of R_1 , R_2 , and I , what is the magnetic field vector $\mathbf{B}$ at the point P that lies at the origin, which is also the center of the two semicircles?
- (b) **(4 points)** This loop is now placed in a uniform magnetic field $\mathbf{B} = B_0 \hat{\mathbf{x}}$ that points everywhere in the $+\hat{\mathbf{x}}$ direction. Discuss briefly whether there is now a net force that acts on the loop (that would accelerate it in some direction), and discuss briefly whether there is now a net torque that acts on the loop (that would rotate the loop about some axis.)
3. **(10 points)** A flat circular non-conducting disk of radius R is covered with a uniform surface charge density σ , and the disk rotates with constant angular frequency ω about an axis that passes perpendicular to and through the disk's center. In terms of R , σ , and ω , find a mathematical expression for the magnitude B of the magnetic field at the center of this rotating disk.
4. Consider two infinite straight cylindrical coaxial conducting shells of outer radius R_1 and inner radius $R_2 < R_1$ such that the outer shell carries a current I_1 to the right and the inner shell carries a current $I_2 < I_1$ to the left as shown here:



The figure on the left shows a cross-section of the two shells when looking down the shells from left to right, so current I_1 is going into the page, current I_2 is coming out of the page.

- (a) **(6 points)** What are the directions of the magnetic field at the points A, B, and C?
- (b) **(6 points)** Assume that these conducting shells are made of a sponge-like elastic conducting material that is capable of expanding, contracting, or stretching under the influence of a force. Discuss briefly whether the inner shell will expand in radius, contract in radius, stretch axially, or contract axially, or do nothing. Similarly discuss briefly what happens to the outer shell. A hint is that you can think of a cylindrical shell (or more generally a wire of finite thickness) carrying a current along its length as the union of many parallel infinitesimal discrete wires all carrying small currents in the same direction.
- (c) **(12 points)** Use Ampere's law to obtain expressions for the magnitudes of the magnetic field $\mathbf{B}(r)$ in the three regimes $R_2 > r \geq 0$, $R_1 > r > R_2$, and $r > R_1$.

Some equations:

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q \mathbf{v} \times \hat{\mathbf{r}}}{r^2}, \quad \mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}, \quad \oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{encl}}, \quad B_{\text{ring center}} = \frac{\mu_0 I}{2R}, \quad \mu_0 \approx 10^{-6}.$$

$$a = \frac{v^2}{r}, \quad \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}, \quad \boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}, \quad \boldsymbol{\mu} = N I \mathbf{A}, \quad \mathbf{A} = A \hat{\mathbf{n}}.$$