

Print your name clearly: \_\_\_\_\_

Signature: \_\_\_\_\_

"I agree to abide by the Duke Community Standard and will neither give nor receive aid during this quiz."

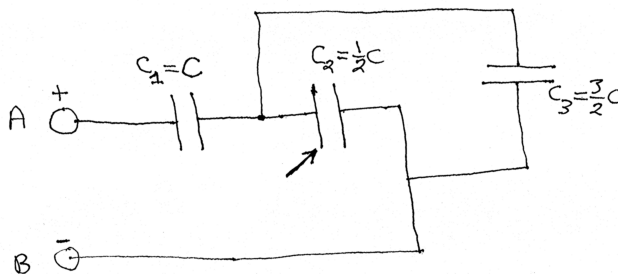
## Physics 162: Quiz 5

Professor Greenside  
Wednesday, February 18, 2015

This 30-minute quiz is closed book and nothing is allowed on your desk except items to write with. Please write all answers on the blank pages handed out in class. Please also write your name and problem number on each page, order the pages in increasing order of problem number, and staple all of your pages together at the end of the quiz.

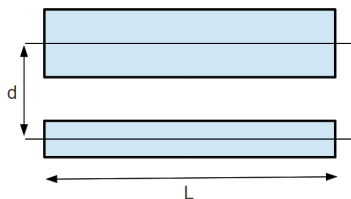
Unless stated otherwise, you need to justify your reasoning, e.g., with brief phrases and drawings; a correct answer without some explanation will get zero credit. If your answers are not easily readable and understandable by the grader, you will lose credit and possibly get no credit.

1. A metal sphere of radius  $R_1 = 2$  cm and initial charge  $Q_{1i} = 1$  nC is brought in contact with a second metal sphere of radius  $R_2 = 4$  cm and initial charge  $Q_{2i} = -4$  nC and then the two spheres are separated by a distance of 100 cm. After separation:
  - (a) **(6 points)** What approximately are the final electric charges  $Q_{1f}$  and  $Q_{2f}$  of the two spheres?
  - (b) **(4 points)** To the nearest power of ten and in units of V/m, what is the largest electric field magnitude  $E$  of the electric field produced by the two separated spheres, and where does that largest magnitude occur in space?
2. Consider the following electronic circuit consisting of three capacitors with capacitances of  $C_1 = C$ ,  $C_2 = C/2$ , and  $C_3 = 3C/2$  where  $C > 0$  is some positive number:



- (a) **(5 points)** In terms of  $C$ , what is the effective capacitance  $C_{AB}$  of the circuit between the points A and B?

- (b) **(5 points)** The positive terminal of a battery with emf  $\mathcal{E}$  is connected to point A and the negative terminal of the battery is connected to point B. After electrostatic equilibrium occurs, and in terms of  $\mathcal{E}$  and  $C$ , what is the charge  $Q_2$  (sign and magnitude) on the plate of capacitor  $C_2$  indicated by the arrow?
- (c) **(5 points)** The three capacitors each have a maximum voltage rating of  $V_{\max}$ . (A capacitor can malfunction electrically and irreversibly if the potential difference across the capacitor approximately exceeds its voltage rating.) If a power supply with variable voltage difference  $\Delta V$  is connected to points A and B and the potential difference  $\Delta V$  is slowly and steadily increased, for what value of  $V$  (in terms of  $V_{\max}$  and  $C$ ) will one of the capacitors first fail, and which of the capacitors will be the first to fail?
- (d) **(5 points)** A battery with emf  $\mathcal{E}$  is connected between points A and B and then a plastic dielectric block with dielectric constant  $\kappa > 1$  is inserted between the plates of capacitor  $C_2$  so as to exactly fill the space between its plates. Determine whether the potential difference across capacitor  $C_1$  increases, decreases, or stays the same after the dielectric block is inserted into  $C_2$ .
- (e) **(5 points)** If you are allowed to connect these three capacitors in any possible way and given that all three capacitors have a maximum rating of  $V_{\max}$ , what is the maximum amount of energy you can store with these capacitors in terms of  $V_{\max}$  and  $C$ ? (Possibly helpful formulas:  $U = Q^2/(2C) = (1/2)CV^2$ .)
3. Two long hollow metal cylindrical shells, open at both ends, of length  $L$  and of radii  $R_1$  and  $R_2$  are placed parallel to one another and separated by a distance  $d$  as shown here (not drawn to scale):



- (a) **(12 points)** Derive and give a mathematical expression for the capacitance  $C$  of this system in terms of  $L$ ,  $d$ ,  $R_1$ , and  $R_2$ .
- (b) **(4 points)** To the nearest power of ten, estimate the capacitance  $C$  in farads F of these parallel cylindrical shells for the parameter values

$$R_1 = 0.1 \text{ cm}, \quad R_2 = 2 \text{ cm}, \quad d = 20 \text{ cm}, \quad \text{and} \quad L = 100 \text{ cm}. \quad (1)$$

- (c) **(4 points)** If the hollow metal cylindrical shells are each replaced by solid metal cylinders of the same dimensions, does the capacitance increase, remain the same, or decrease?

Note: the cylinders are so long that you can ignore end effects, and that they are so far apart that you can ignore polarization of one cylinder by the other. Remember that to get the capacitance of two conductors, put a charge  $Q$  on one conductor and a charge  $-Q$  on the other cylinder then calculate the resulting electric potential difference  $-\int \mathbf{E} \cdot d\mathbf{l}$  between the conductors along some path. Also recall that Gauss's law is  $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{encl}}/\epsilon_0$  where  $K = 1/(4\pi\epsilon_0)$ , and observe that  $e^3 \approx 20$  where  $e \approx 2.7$  is the base of the natural logarithm.