Physics 162: Quiz 4
Professor Greenside
Wednesday, February 11, 2015

This 30-minute quiz is closed book and nothing is allowed on your desk except items to write with. Please write all answers on the blank pages handed out in class. Please also write your name and problem number on each page, order the pages in increasing order of problem number, and staple all of your pages together at the end of the quiz.

Unless stated otherwise, you need to justify your reasoning, e.g., with brief phrases and drawings; a correct answer without some explanation will get zero credit. If your answers are not easily readable and understandable by the grader, you will lose credit and possibly get no credit.

1. (8 points) A rigid neutral immobile plastic square with side length $L$ has point particles placed at its vertices with charges $q_1$, $q_2$, $q_3$, and $q_4$ such that $q_1 + q_2 + q_3 + q_4 > 0$. A long thin frictionless neutral thread is passed through the center of the square and perpendicular to the plane of the square. At a distance $D$ from the square, a tiny bead is placed on the thread with speed $v$, mass $m$, and positive charge $Q$ such that the bead is moving towards the square.

If the bead comes to a stop a distance $d < D$ from the square, what was the speed $v$ of the bead in terms of given parameters?

2. (8 points) A battery with a designated voltage $\Delta V$ is a device that maintains a constant potential difference of $\Delta V$ between any two metal objects that are connected to the battery’s two metal connectors (which are called the poles or terminals of the battery). Consider a simple home-made parallel-plate capacitor consisting of two 20 cm $\times$ 20 cm sheets of aluminum foil separated by a sheet of paper of 0.1 mm thickness. You then connect wires from the two sheets of foil to the poles of a 1.5 V battery, which causes charges of $Q$ and $-Q$ to appear on the two aluminum sheets such that a potential difference of 1.5 V exists between the sheets. Estimate to the nearest power of ten the magnitude of the charge $|Q|$ on each sheet in coulombs. (For comparison, a rod charged by friction will have a total charge of magnitude $1 - 100 \text{nC}$ where one nC (nanocoulumb) is $10^{-9}$ C.)
3. **(10 points)** An infinite non-conducting cylindrical shell of negligible thickness, radius \( R \), and uniform negative surface charge density \( \sigma_1 < 0 \) is placed parallel to and above an infinite plane of uniform positive surface charge density \( \sigma_2 > 0 \) so that the axis of the shell is a distance \( D \) above the plane (with \( D > R \)). In cross-section, the cylinder and plane look like this, with the shell and plane being perpendicular to the page:

![Diagram of a cylindrical shell and a plane](image)

What is the potential difference \( V(B) - V(A) \) between the plane and the part of the shell closest to the plane? (Hint: first calculate the total electric field between points A and B and then evaluate some line integral.)

4. A hollow non-conducting cylindrical shell of negligible thickness has finite length \( L \), radius \( R \), and charge \( Q \) uniformly distributed over its surface. By treating this shell as the union of many infinitesimal rings:

   (a) **(12 points)** Find and give an explicit expression (with any integrals fully evaluated) for the electric potential \( V \) at the point on the axis that lies at the center of the shell.

   (b) **(3 points)** Estimate \( V \) in volts to the nearest power of ten for the values \( L = 30 \) cm, \( R = 1.5 \) cm and \( Q = 10^{12} e \), i.e., for a rubber shell that is about a foot long, about an inch in diameter, and that has been rubbed vigorously with cat fur.

   (c) **(2 points)** In the limit that the cylindrical shell becomes infinitely long, \( L \to \infty \), while the surface charge density remains constant, Gauss’s law tells us that the electric field everywhere inside the shell is zero. According to your result in (a), in this limit \( L \to \infty \) (but with \( Q/L \) constant), what is the potential on the axis of this uniformly charged shell?

Some possibly useful information:

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\epsilon_0 = \frac{1}{4 \pi K}, \quad \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{encl}}}{\epsilon_0}, \quad V(B) - V(A) = -\int_A^B \mathbf{E} \cdot d\mathbf{l}, \quad V_{\text{point}}(d) = \frac{KQ}{d}.
\]

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\ln(10) \approx 2.3, \quad \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left( x + \sqrt{x^2 + a^2} \right), \quad \int \frac{x \, dx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2}.
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