

Print your name clearly: _____

Signature: _____

"I agree to abide by the Duke Community Standard and will neither give nor receive aid during this quiz."

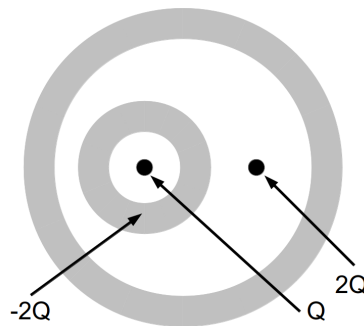
Physics 162: Quiz 3

Professor Greenside
Wednesday, February 4, 2015

This 30-minute quiz is closed book and nothing is allowed on your desk except items to write with. Please write all answers on the blank pages handed out in class. Please also write your name and problem number on each page, order the pages in increasing order of problem number, and staple all of your pages together at the end of the quiz.

Unless stated otherwise, you need to justify your reasoning, e.g., with brief phrases and drawings; a correct answer without some explanation will get zero credit. If your answers are not easily readable and understandable by the grader, you will lose credit and possibly get no credit.

1. **(8 points)** Using a symmetry argument and Gauss's law $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{encl}}/\epsilon_0$, obtain an expression for the electric field vector $\mathbf{E}$ at a point P that is a distance d from an infinitely long, straight, uniformly charged line with linear charge density λ .
2. A neutral spherical conducting outer shell contains a point particle with positive charge $2Q$, a charged conducting spherical shell with charge $-2Q$, and there is a point particle with positive charge Q at the center of the inner shell, as shown here:



- (a) **(12 points)** For each of the four surfaces (inner surface of the inner shell, outer surface of the inner shell, inner surface of the outer shell, the outer surface of the outer shell), deduce and give the total charge on that surface and discuss briefly whether the corresponding surface charge density σ is spatially uniform.
- (b) **(6 points)** Assuming that all four objects in the above figure are floating freely in space and so are free to move, and assuming that all non-electrical forces like gravity can be ignored, explain briefly in which direction each item will start to move, or explain why the object will remain in place.

3. Contrary to all modern evidence, members of the still extant Flat Earth Society believe that the Earth is flat. In this problem, you use the gravitational version of Gauss's law to estimate the thickness of a flat Earth that would produce the same surface acceleration $g \approx 10 \text{ m/s}^2$ as our spherical Earth.

As background, observe that since gravity also obeys an inverse square law (except near neutron stars and black holes), Gauss's law also holds for static gravitational fields, in which case it takes the form

$$\int_S \mathbf{E}_g \cdot d\mathbf{A} = \frac{M_{\text{encl}}}{\epsilon_G}. \quad (1)$$

The surface integral on the left side is the gravitational flux of a gravitational vector field $\mathbf{E}_g$ through an arbitrary closed mathematical surface S (a Gaussian bubble), $d\mathbf{A}$ is an infinitesimal area vector corresponding to a small planar area dA on the surface of S , the constant ϵ_G has the value $\epsilon_G = 1/(4\pi G)$ where Newton's gravitational constant G has the approximate value $G \approx 10^{-10} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, and M_{encl} is the total mass in kg enclosed by the bubble S . By analogy to how the electric field $\mathbf{E} = \mathbf{F}/q$ is defined via the electric force $\mathbf{F}$ acting on some small probe charge q , the gravitational field $\mathbf{E}_g(x, y, z)$ at a particular point $P = (x, y, z)$ is defined to be the ratio $\mathbf{E}_g = \mathbf{F}_g/m$ where $\mathbf{F}_g$ is the total gravitational force acting on some tiny probe of small mass m at P . Gravitational fields therefore have units of F/m or force/mass or acceleration, and we could call them "acceleration fields" instead.

- (a) **(12 points)** Assuming that the Earth is an infinitely wide flat slab of finite thickness H with a uniform mass density ρ in units of kg/m^3 , use the gravitational version of Gauss's law, Eq. (1), to obtain a mathematical expression in terms of G , H , and ρ for the gravitational field vector $\mathbf{E}_g$ at all points above the surface of the flat Earth. Note: make sure to explain your choice of Gaussian bubble S and explain how you evaluate the left and rights sides of Eq. (1).
- (b) **(6 points)** Assuming that the gravitational field E_g near the flat Earth's surface has the same value as Earth's surface acceleration $g \approx 10 \text{ m/s}^2$, and using for the value of ρ the known average mass density of the Earth, $\rho \approx 6,000 \text{ kg/m}^3$, estimate to the nearest power of ten the dimensionless ratio H/D of the thickness H of the flat Earth to Earth's diameter $D \approx 13,000 \text{ km}$. (As usual, round numbers to the nearest power of ten *before* calculating to avoid having to multiply or divide any digits; unnecessary arithmetic in your answer will cost you points.)

True or False and Why Questions (3 points each)

For each question, indicate whether it is true or false and briefly explain your reasoning.

1. **T / F** Gauss's law can be used to calculate the electric field at any point outside of a non-conducting cubical surface



that has been coated with a uniform surface charge density $\sigma \neq 0$. (There is nothing inside the cubical surface.)

2. **T / F** For the above non-conducting cubical surface that has been coated with a uniform surface charge density $\sigma \neq 0$, the electric field is everywhere zero inside the surface.
3. **T / F** If a point particle with positive charge $Q > 0$ lies a distance d from an infinite plane, the electric flux through the plane has magnitude $Q/(2\epsilon_0)$.