Two comments here, one an illustration of how to use the general binomial theorem:

\[
(1 + x)^r = 1 + \frac{r}{1!}x + \frac{r(r-1)}{2!}x^2 + \frac{r(r-1)(r-2)}{3!}x^3 + \cdots \quad |x| < 1
\]

and how to compute and use unit vectors related to Coulomb's law.

Assume you have a quadrupole of charges, which is a neutral arrangement of 4 charges that also has no net electric dipole:

There are point charges \( q, q, -q, -q \) at coordinates \( (0, d), (d, 0), (-d, 0), (0, -d) \) respectively and a positive charge \( Q \) at coordinate \( (L, 0) \) with \( L \gg d \)

What is the net force of the quadrupole on \( Q \)?
You should verify that, according to Coulomb's law, the exact total force $\vec{F}_{\text{tot}}$ acting on $Q$ is

$$\vec{F}_{\text{tot}} = \vec{F}_{\text{A on Q}} + \vec{F}_{\text{B on Q}} + \vec{F}_{\text{C on Q}} + \vec{F}_{\text{D on Q}}$$

$$= (\vec{F}_{\text{A on Q}} + \vec{F}_{\text{B on Q}}) + \left(\frac{K_8 Q}{(1-d)^2} + \frac{K_8 Q}{(2+d)^2}\right)$$

$$= \left[2 \cdot \frac{K_8 Q}{d^2 + L^2} \cos \gamma - \frac{K_8 Q}{(2-d)^2} - \frac{K_8 Q}{(2+d)^2}\right] \hat{y}$$

Since $\vec{F}_{\text{A on Q}}$ is a vector like this and $\vec{F}_{\text{B on Q}}$ has same horizontal component but opposite vertical component,

so $\vec{F}_{\text{A on Q}} + \vec{F}_{\text{B on Q}} = c \hat{x}$ for some positive constant $c$.

From the geometry,

we see that

$$\cos \gamma = \frac{1}{\sqrt{p^2 + L^2}}$$
So total force of quadrupole on \( Q \) (which is negative)

\[
\vec{F}_{\text{tot}} = \hat{x} \left[ \frac{2K_2 Q \cdot L}{(d^2+L^2)^{3/2}} - \frac{K_8 Q}{(L-d)^2} - \frac{K_8 Q}{(L+d)^2} \right]
\]

(4)

Let's apply binomial approximation to term in brackets.

Observe \( L \gg d \) so also \( L^2 \gg d^2 \), we first write each power to \( (1+\varepsilon)^x \) with \( \varepsilon \) small compared to \( 1 \) and all power appearing in numerators. So

\[
\frac{1}{(d^2+L^2)^{3/2}} = \frac{1}{L^3 \left( 1 + \frac{d^2}{L^2} \right)^{3/2}} = \frac{1}{L^3} \left( 1 + \frac{d^2}{L^2} \right)^{-3/2}
\]

\[
\frac{1}{(L-d)^2} = \frac{1}{L^2 \left( 1 - \frac{d}{L} \right)^2} = \frac{1}{L^2} \left( 1 - \frac{d}{L} \right)^{-2}
\]

\[
\frac{1}{(L+d)^2} = \frac{1}{L^2 \left( 1 + \frac{d}{L} \right)^2} = \frac{1}{L^2} \left( 1 + \frac{d}{L} \right)^{-2}
\]

Substituting into (4) above, factoring out some common coefficients we get

\[
\vec{F}_{\text{tot}} = \hat{x} \cdot \frac{K_8 Q}{L^2} \cdot \left[ 2 \left( 1 + \frac{d^2}{L^2} \right)^{-3/2} - (1-\frac{d}{L})^{-2} - (1+\frac{d}{L})^{-2} \right]
\]

(3)
Let's apply the binomial theorem to (1), (2), and (3) in turn. We have to retain powers of $\frac{d}{L}$ up to 2nd order, $(\frac{d}{L})^2$, to get the first non-zero power, which then give us the leading approximation. We have:

1. \[2(1 + \frac{d^2}{L^2})^{3/2} \approx 2 \left[ 1 + \frac{-3}{2} \left( \frac{d}{L} \right)^2 + \frac{3 \cdot 5 / 2}{1 \cdot 2} \left( \frac{d}{L} \right)^4 + \ldots \right] \]

we can drop (ignore) the $(\frac{d}{L})^4$ since it is much smaller than $(\frac{d}{L})^2$

\[\approx 2 \left[ 1 - \frac{3}{2} \left( \frac{d}{L} \right)^2 \right] \]

2. \[-\left(1 - \frac{d}{L}\right)^2 = - \left[ 1 + \frac{2 \cdot 3}{1} \left( \frac{d}{L} \right) + \frac{(-1) \cdot 2 \cdot (-3)}{1 \cdot 2} \left( \frac{d}{L} \right)^2 + \text{cubic} \ldots \right] \]

\[\approx -1 + \frac{3}{2} \left( \frac{d}{L} \right)^2 - 3 \left( \frac{d}{L} \right)^2 \]

3. \[-\left(1 + \frac{d}{L}\right)^2 \approx -1 + 2 \left( \frac{d}{L} \right) - 3 \left( \frac{d}{L} \right)^2 \]

Adding (1) + (2) + (3) gives

\[2(1 + \frac{d^2}{L^2})^{3/2} - \left(1 - \frac{d}{L}\right)^2 - \left(1 + \frac{d}{L}\right)^2 \]

\[\approx 2 - 3 \left( \frac{d}{L} \right)^2 - 1 - 2 \left( \frac{d}{L} \right) + 3 \left( \frac{d}{L} \right)^2 \]

\[\approx -3 \left( \frac{d}{L} \right)^2 + 2 \left( \frac{d}{L} \right) - 3 \left( \frac{d}{L} \right)^2 \]
We conclude that

\[ F_{\text{tot}} \approx \sum \frac{k_\text{B} q^2}{l^2} \left( -q(x) \right) = -\frac{9k_\text{B} q d^2}{l^4} \]

This is a force that decreases as \( 1/l^4 \) with increasing \( l \).

The minus sign means the total force on \( Q \) due to the quadrupole acts to the left, i.e., the quadrupole and charge \( Q \) attract each other, although weakly at long distances, \( L \gg d \). This is in agreement with our qualitative reasoning based on the fact that charge \( Q \) at \( (x, y) \) is closer to \( Q \) than \( Q \) to \( Q \) so dominates the interaction.