Assignment 6
Solutions

1. The number of circles that each electron makes per second:

$$N = \frac{3\cdot10^8\ m/s}{107.5\ m} = 2.791\times10^6\ s^{-1}$$

The charge per second passing any given point:

$$I = \frac{Q}{\Delta t} = N\cdot Ne\cdot e$$
$$= (2.791\times10^6\ s^{-1})(2\times10^{11})(1.6\times10^{-19}\ C)$$
$$= 0.09\ A \approx 100\ mA$$

If all 2×10^{11} 1-GeV electrons transfer energy into your hand:

$$E = (2\times10^{11})(1.0\times10^9)(1.6\times10^{-19})\ J = 32\ J$$

This energy is close to the kinetic energy of a standard 0.4 kg football flying at about 13 m/s.

2. According to the definition of current:

$$I = \frac{dQ}{dt} = \frac{\Delta Q}{\Delta t}$$

If the cone rotates by 2π, then:

$$\Delta Q = A \cdot \sigma$$

where A is the lateral surface area of the cone. To get A, we flatten the cone like shown below:

So, $A = \pi h^2 \cdot \frac{\theta}{2\pi} = \pi h^2 \frac{2\pi R/h}{2\pi} = \pi R h$

in which $h = \sqrt{H^2 + R^2}$

Therefore:

$$A = \pi R \sqrt{H^2 + R^2}$$
$$\Delta Q = \pi R \sigma \sqrt{H^2 + R^2}$$

and $\Delta t = \frac{2\pi}{\omega}$

$$\Rightarrow I = \frac{\Delta Q}{\Delta t} = \frac{\pi R \sigma \sqrt{H^2 + R^2}}{2\pi/\omega} = \frac{1}{2} R \sigma \omega \sqrt{H^2 + R^2}$$

3. We know that the current density is $J = Nev \Rightarrow v = \frac{J}{Ne}$. So our goal is to determine J. It is given by:

$$J = \sigma E = \frac{1}{\rho}\frac{V}{L}$$

Alternatively, we can find J from:

$$J = \frac{I}{A} = \frac{V/R}{A} = \frac{V}{A}\frac{1}{\rho L/A} = \frac{V}{\rho L}$$

The drift velocity is therefore (the factor of 2 in the N here comes from the fact that there are both Na^+ and Cl^- ions):

$$v = \frac{J}{Ne} = \frac{V}{\rho L N e} = \frac{12V}{(0.15\,\Omega m)(2m)(2.3\cdot 10^{26} m^{-3})(1.6\cdot 10^{-19} C)}$$

$$= 2.5 \times 10^{-7}\ m/s \approx 3 \times 10^{-7}\ m/s$$

4. (a) The resistance of the seven wires together is:

$$R = \frac{\rho L}{A} = \frac{(3\times 10^{-8}\,\Omega m)(3\times 10^{6} m)}{7\cdot\pi(3.63\times 10^{-4} m)^2} = 3.1 \times 10^{4}\,\Omega$$

Adding seven resistors in parallel would give the same answer.

(b) Our goal is to obtain a rough upper bound on the resistance of the ocean path, and then see if this is smaller than the answer to part (a). As stated, we'll take the electrodes to be spheres with a radius of 0.1 m (Any factors of order 1 will be irrelevant). Let's imagine the path of the current to be roughly a hemisphere expanding out from one sphere, then a tube with a large cross section, and then a hemisphere tapering down to the other sphere (Even if the end parts are more conical than hemispherical, we'll still get the same order of magnitude). For the tubular middle part,

it doesn't matter much what cross sector we pick, if our only goal is to obtain an upper bound on the resistance. Even if the cross section is just a kilometer in diameter, the resistance (using a length of 3000 km):

$$R = \frac{\rho L}{A} = \frac{(0.25\,\Omega\,m)(3\times10^6\,m)}{\pi(10^3\,m)^2/4} \approx 1\,\Omega.$$

which is negligible compared with the resistance of the cable.

What is the resistance of the hemispheres? If $d \approx 0.1\,m$ is their radius, then from dimensional analysis we expect the resistance to be roughly ρ/d, in order of magnitude. ρ/d equals $(0.25\,\Omega\,m)/(0.1\,m) = 2.5\,\Omega$. This is negligible compared with the cable's resistance. Even if the electrode's size was on the order of a millimeter the hemisphere's resistance would still only be about 250 ohms, which is again negligible compared with the cable's resistance!

5. (a) The expression that gives the resistance of a resistor in terms of its physical properties is:

$$R = \rho \frac{L}{A}$$

We simply substitute ρ by a temperature depending $\rho(T)$

$$R = \frac{L}{A}\rho_0[1+\alpha(T-T_0)]$$

The resistance of two temperature depending resistors connected in series is then:

$$R = R_1 + R_2 = \frac{1}{A}\left(L_1\rho_1[1+\alpha_1(T-T_0)] + L_2\rho_2[1+\alpha_2(T-T_0)]\right)$$
$$= \frac{1}{A}\left(L_1\rho_1 + L_2\rho_2 + (L_1\rho_1\alpha_1 + L_2\rho_2\alpha_2)(T-T_0)\right).$$

Therefore if $L_1\rho_1 d + L_2\rho_2 d_2 = 0$ is satisfied, there would be no temperature dependence in the expression.

(b) Plugging in the values:

$$L_1(1.7\times10^{-8}\,\Omega\cdot m)(3.9\times10^{-3}\,K^{-1}) + L_2(3500\times10^{-8}\,\Omega\cdot m)(-0.5\times10^{-3}\,K^{-1}) = 0$$

$$\Rightarrow L_{Cu}/L_{graphite} = 2.6\times10^{2},$$

which seems to be not very practical.

6. For the top, middle, and bottom segments the subscripts "top", "mid", and "bot" are used.

(a) Since current is conserved, $I_{top} = I_{mid} = I_{bot} = 10\,A$.

(b) The current density is $J = I/A = I/(\pi R^2)$. Thus:

$$J_{top} = J_{bot} = \frac{10\,A}{\pi(0.003\,m)^2} = 3.18\times10^{6}\,A/m^2$$

$$J_{mid} = \frac{10\,A}{\pi(0.0001\,m)^2} = 1.27\times10^{9}\,A/m^2$$

(c) The electric field is $E = J/\sigma$. Thus,

$$E_{top} = E_{bot} = \frac{J_{top}}{\sigma} = \frac{3.18\times10^{6}\,A/m^2}{3.5\times10^{7}\,\Omega^{-1}m^{-1}} = 0.0909\,V/m$$

$$E_{mid} = \frac{J_{mid}}{\sigma} = \frac{1.27\times10^{9}\,A/m^2}{3.5\times10^{7}\,\Omega^{-1}m^{-1}} = 0.364\,V/m$$

(d) The drift speed is $v_d = J/(ne)$. Thus:

$$(v_d)_{top} = (v_d)_{bot} = \frac{J_{top}}{ne} = \frac{3.18\times10^{6}\,A/m^2}{(6.0\times10^{28}\,m^{-3})(1.60\times10^{-19}\,C)} = 3.31\times10^{-4}\,m/s$$

$$(v_d)_{mid} = \frac{J_{mid}}{ne} = \frac{1.27\times10^{9}\,A/m^2}{(6.0\times10^{28}\,m^{-3})(1.60\times10^{-19}\,C)} = 1.33\times10^{-3}\,m/s$$

(e) The electron current is $i = Ne/\Delta t = I/e$. Because $I = I_1 = I_2 = I_3 = 10\,A$:

$$\left(\frac{Ne}{\Delta t}\right)_{top} = \left(\frac{Ne}{\Delta t}\right)_{mid} = \left(\frac{Ne}{\Delta t}\right)_{bot} = \frac{10\,A}{1.60\times10^{-19}\,C} = 6.25\times10^{19}\,s^{-1}$$

7. (a) The expression for collision time τ is $\tau = \tau_0(1+\alpha E)$, where $\tau_0 > 0$ and α are constants. If the magnitude of the electric field E increases, the electrons will accelerate under greater acceleration, and the collision time will reduce. Therefore, constant α should be negative.

(b) The expression for current I in terms of electron charge e, electron density n_e, cross section area A, and drift speed of electrons V_d is:

$$I = e n_e V_d A,$$

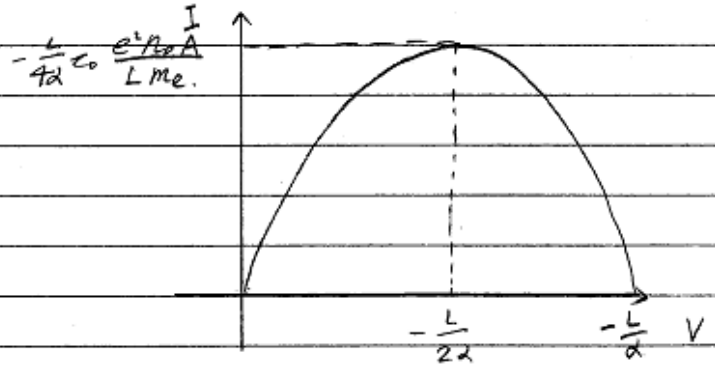
where $V_d = (\frac{eE}{m_e})\tau$, and $E = \frac{V}{L}$. Plugging in those values:

$$I = \frac{e^2 n_e A}{L m_e} \cdot V \cdot \tau.$$

Plugging in the V dependent expression of τ:

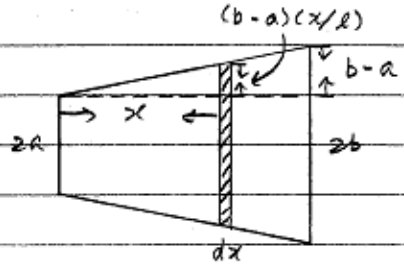
$$I = \frac{e^2 n_e A}{L m_e} \tau_0 V(1+\alpha\frac{V}{L}).$$

Qualitatively, the I-V curve of current I versus potential V should look like:



8. Let the length of the rod be ℓ. Then the resistance of the cylindrical rod is $\rho L/A = \rho L/(\pi a^2)$.

Now consider the tapered cone. The resistance of a cross-sectional disk is $\rho L/A = \rho dx/(\pi r^2)$, where $r = a + (b-a)(x/\ell)$ from similar triangles shown below.



So $dx = dr\ell/(b-a)$, and we have:

$$R = \int_0^\ell \frac{\rho\, dx}{\pi r^2} = \frac{\rho}{\pi}\frac{\ell}{b-a}\int_a^b \frac{dr}{r^2} = \frac{\rho}{\pi}\frac{\ell}{b-a}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho\ell}{\pi ab}$$

This is a/b times the resistance of the cylindrical rod, as desired.

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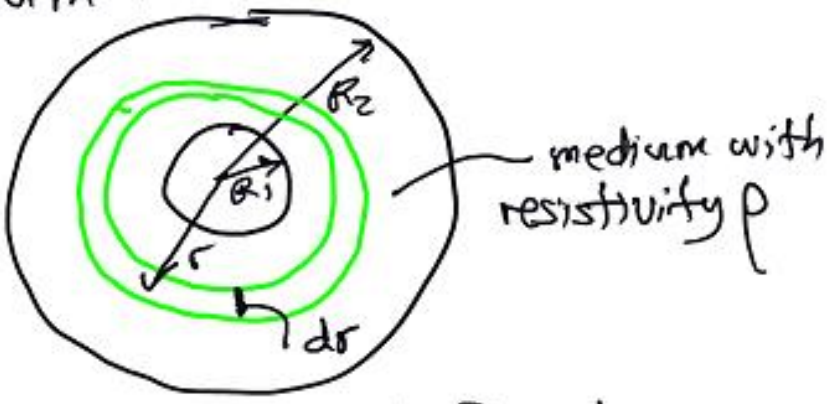
(a)

In cross-section, the two shells filled with a resistive medium look like this:



To get the resistance, apply the formula

$$R = \rho \frac{L}{A}$$

to an infinitesimal shell of resistive material of radius r and thickness dr, as shown in green above. Here L = dr (since current must flow radially by symmetry, transverse to the thin shell) and $A = (2\pi r)L$ so

$$dR = \text{resistance of thin shell of radius } r, \text{ thickness } dr = \rho \frac{dr}{(2\pi r)L}$$

$$R = \int_{R_1}^{R_2} dR$$

Total resistance is therefore

$$R = \int_{R_1}^{R_2} \rho \frac{dr}{(2\pi r L)}$$

$$= \frac{\rho}{2\pi L} \ln\left[\frac{R_2}{R_1}\right]$$

Effectively, the successive shells act like resistances in series and so add.

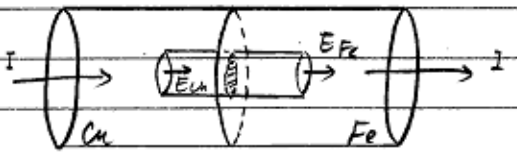
(b) For the given parameters, the resistance of the resistive medium is

$$R = \frac{30.0\,\Omega\cdot m}{2\pi[0.5m]} \ln\left[\frac{2.50\,cm}{1.50\,cm}\right]$$

$$\approx 4.88\,\Omega$$

and by Ohm's law, the current produced by an emf of 10V is

$$I = \frac{\mathcal{E}}{R} = \frac{10.0V}{4.88\Omega} \approx 2.05A$$



1a) We place a Gaussian bubble like a cylinder across the weld as shown. If E_{Cu} is the magnitude of the electric field in Cu part of the wire and E_{Fe} is that in Fe part of the wire, and noting that the $\vec{E}$ field point in the direction of the current and areas point outward by convention, we have:

$$\oint \vec{E}\cdot d\vec{A} = \int_{\text{left cap}} \vec{E}\cdot d\vec{A} + \int_{\text{cylinder shell}} \vec{E}\cdot d\vec{A} + \int_{\text{right cap}} \vec{E}\cdot d\vec{A}$$

$$= -E_{Cu}A + 0 + E_{Fe}A$$

$$= A(E_{Fe} - E_{Cu})$$

By Gauss's law, it must equal $\frac{Q_{enclosed}}{\varepsilon_0}$. This implies $Q_{enclosed}$, the charge on the weld within area A, is nonzero, provided that $E_{Cu} \neq E_{Fe}$.

So we now need to figure out how E_{Cu} and E_{Fe} differ. If we consider a segment of just the Cu wire of length L with a current I passing through the wire, we have from Ohm's Law:

$$V = EL = IR = (\rho \tfrac{L}{A})I \Rightarrow E = \frac{\rho I}{A}$$

The electric field is proportional to ρ, I, inversely proportional to A. Substituting into Gauss's Law equation

$$\frac{Q_{enclosed}}{\varepsilon_0} = A(E_{Fe} - E_{Cu}) = (\rho_{Fe} - \rho_{Cu})I$$

Since $\rho_{Fe} > \rho_{Cu}$, there must be a positive charge on the weld.

(b) To estimate the amount of charge on the weld:

$$Q_{enclosed} = (\rho_{Fe} - \rho_{Cu}) I \xi$$
$$= (1\times10^{-7} - 2\times10^{-8}) \times 10 \times 10^{-11} \ (C)$$
$$\approx 10^{-17} C \approx 10^{2} e$$

So about 10^2 electrons will accumulate on the weld for a current of order an amp, which is extremely tiny to be experimentally measured.