

Physics 162 Assignment 9

Made available: Wednesday, March 25, 2015

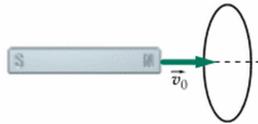
Due: Sunday, March 29, 2015 by 9 pm

1. Problem 79 on page 1002 of Knight 3e. You should get the answer:

$$\Delta V = \frac{\mu_0 v I}{2\pi} \ln \left(\frac{d+l}{d} \right). \quad (1)$$

Note how this answer depends reasonably on parameters, e.g., a bigger current I or a bigger speed v causes a bigger potential difference ΔV , and a rod of vanishing length, $l \rightarrow 0$, corresponds to zero potential difference.

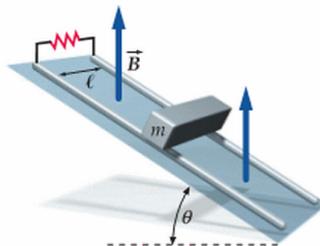
2. A bar magnet moves with constant velocity \mathbf{v}_0 along the axis of a loop as shown here:



- Make a qualitative graph of the magnetic flux Φ_m through the loop as a function of time, and indicate on your graph when the magnet is halfway through the loop by designating this time as t_1 . (Assume that the normal to the flat surface bounded by the loop points to the right.)
- Make a graph of the induced current $I(t)$ in the loop as a function of time. (Choose the positive direction for the current to be clockwise as viewed along the axis from the left.)
- Repeat part (b) for the passage through the loop of a hypothetical magnetic monopole, a point particle that acts like an isolated north pole of a magnet. (Your answer to Problem 1 of Assignment 3 should be useful here.) From your answers, discuss briefly whether one could distinguish in principle between a monopole and a dipole passing through the loop by monitoring the current of such a loop as a function of time.

Note: we are making a naive assumption, that any mysterious particles arrive along the axis of the loop. More realistically, particles could pass through the loop arriving from any possible direction, and a dipole might possibly be rotating about some unknown axis with some unknown angular frequency as it passes through the loop. For some optional extra credit, can you still distinguish a dipole from a monopole under these more general circumstances?

3. As shown in this figure



a conducting rod with mass m can slide without friction down two parallel conducting rails that are separated by a distance L and connected by a resistor with resistance R . The rails are attached to a long inclined plane that makes an angle θ with the horizontal, and the entire system is immersed in a uniform magnetic field of strength B that is pointed vertically upwards. The resistances of the bar and rails are so small that they can be neglected.

- (a) Show that there is a retarding force F directed up the incline plane with magnitude

$$F = \frac{B^2 L^2 v \cos^2(\theta)}{R}. \quad (2)$$

- (b) Show that the rod reaches a terminal (asymptotic) speed of

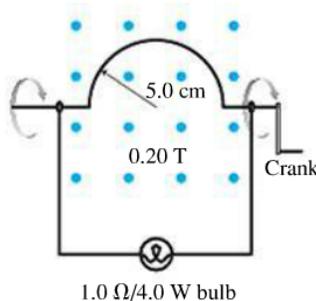
$$v_t = \frac{mgR \sin(\theta)}{B^2 L^2 \cos^2(\theta)}. \quad (3)$$

- (c) To the nearest power of ten, estimate the magnitude of the terminal speed for $m = 1 \text{ kg}$, $g \approx 10 \text{ m/s}^2$, $B = 0.1 \text{ T}$, $L = 10 \text{ cm}$, $R = 1 \text{ k}\Omega$ and $\theta = \pi/4$. (Do *not* use a calculator when estimating and show explicitly how you did your estimation.)

4. Problem 58 on page 1000 of Knight 3e.

You should get answers for (a) and (b) of respectively $E_{\text{max}} \approx 0.94 \text{ V/m}$ and $B = 10 \text{ T}$.

5. Your camping buddy has an idea for lighting the inside of your tent. She has brought a powerful and heavy horseshoe magnet from a surplus store that creates a 0.20 T magnetic field between its two pole tips that are 10 cm apart. Her idea is to build a hand-cranked generator that looks like this:



- (a) If your buddy turns the crank with constant frequency f , show that the induced current $I(t)$ as a function of time is given by

$$I = \frac{\pi^2 r^2 B f}{R} \sin(2\pi f t), \quad (4)$$

if the semicircle is at its highest point at $t = 0$.

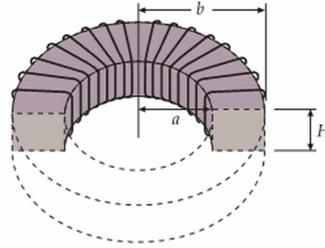
- (b) For a 1Ω 4-W bulb, with what frequency will your buddy have to turn the handle for the maximum current to fully light the bulb? Is this feasible, so that your buddy had a “bright” idea?

6. Just as electric fields store an amount of energy per unit volume given by $u_E = U_E/V = (\epsilon_0/2)E^2$ (see Eq. (29.28) on page 855 of Knight 3e), magnetic fields store an amount of energy per unit volume $u_B = U_B/V = B^2/(2\mu_0)$. Given this, show that the magnetic energy per unit length for a long cylindrical wire that has a radius R and that carries a current I that is uniformly distributed over its cross-sectional area is given by

$$\frac{U}{L} = \frac{\mu_0}{16\pi} I^2. \quad (5)$$

(Note how this expression does not depend on the radius of the wire, can you see intuitively why?)
 Then estimate U/L to the nearest power of ten for the values $R = 2.0$ cm and $I = 80$ A.

7. Consider this toroid with rectangular cross section



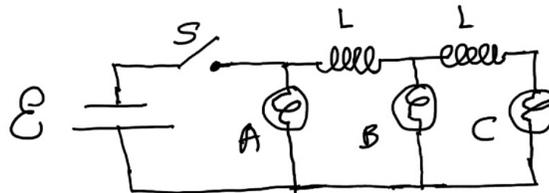
which has an inside radius a , outside radius b , height H , and total number of turns N .

- Use Ampere's law to find an expression for the magnitude of the magnetic field anywhere inside and outside the toroid.
- Show that the inductance \mathcal{L} of this toroid is given by

$$\mathcal{L} = \frac{\Phi}{I} = \frac{\mu_0 N^2 H}{2\pi} \ln\left(\frac{b}{a}\right). \quad (6)$$

Such toroids are often preferred on circuit boards to inductances based on solenoids since their magnetic fields are confined within the toroidal region and so interfere less with other parts of the circuit.

8. A circuit consists of two identical inductors of negligible resistance and of three identical lightbulbs A, B, and C connected to a battery and switch S as shown here:



- After being opened for a long time, the switch S is closed. What are the relative brightnesses of the three bulbs immediately after the switch is closed, from brightest to dimmest?
- After the switch S is closed for a long time, what are the relative brightnesses of the three lightbulbs, from brightest to dimmest?
- The switch is now opened after being closed for a long time. What are the relative brightnesses of the three bulbs immediately after opening the switch, from brightest to dimmest?
 Remember that inductors have the property of preserving the currents flowing through them over short periods of time...

9. Solve Problem 74 on page 1001 of Knight 3e.

10. Time to Complete This Assignment

To one significant digit, please give the time in hours that it took you to complete this homework assignment.

11. Optional Challenge Problems

- (a) Challenge problem 9.1: A charge Q is uniformly distributed over a thin insulating ring of radius R and mass m . To what final angular frequency ω will the ring be accelerated by turning on a uniform magnetic field of strength B that is perpendicular to the plane of the ring?

- (b) Challenge problem 9.2: Consider and answer two variations of problem 3 above, which concerns a metal bar sliding down a ramp in the presence of a uniform magnetic field.
 - i. The resistor is replaced by an inductor with self-inductance L . Describe the resulting motion of the bar after the bar is released at rest, a height h above the ground.

 - ii. The resistor is replaced with a capacitor with capacitance C . Again discuss the resulting motion of the bar after the bar is released at rest, a height h above the ground.