

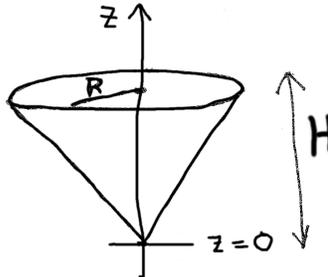
Physics 162 Assignment 6

Made available: Sunday, February 15, 2014

Due: Sunday, February 22, 2015, by 10 AM.

Problems

1. In Duke's Free Electron Laser (the device lives inside the building with blue-slate roofs behind the Physics building), a 1.0 GeV electron accelerator ($1 \text{ GeV} = 10^9 \text{ eV}$) sends about 2×10^{11} electrons traveling around the machine in a closed path of length 107.5 m at effectively the speed of light. Show that the current of this electron beam is $I \approx 100 \text{ mA}$ (but give your answer to two significant digits).
If you stopped this electron beam by putting your hand in its way (so all 10^{11} 1 GeV electrons transfer their energy into your hand), would your hand be badly hurt? Justify your answer quantitatively. Hint: the specific heat of water is about $4.2 \text{ J}/(\text{kg K})$, so how much would the energy of the beam, if deposited fully in your hand, raise the temperature of your hand?
2. An open cone of base radius R and height H has a uniform surface charge density σ and rotates with an angular frequency ω about the cone's axis (the z -axis).



Show that the total current associated with this rotating cone is

$$I = \frac{1}{2} \omega \sigma R \sqrt{R^2 + H^2}. \quad (1)$$

3. The resistivity of seawater is about $\rho \approx 0.25 \Omega \cdot \text{m}$ (that is units of ohm-meters). The charge carriers are mainly Na^+ and Cl^- ions, each with a number density of about $3 \times 10^{26} \text{ m}^{-3}$. If a 2-m long plastic tube is filled with seawater and then a 12 V battery is connected to electrodes at each end of the tube, show that the resulting average drift speed of each ion is about $3 \times 10^{-7} \text{ m/s}$ (but you should give your answer to two significant digits). Note how this drift speed is substantially smaller than that of electrons in a wire.
Note: for the potential biophysics majors in the class, this problem is more relevant than currents in metal wires for understanding how currents flow in organisms, since blood and the cytosol of cells are similar to seawater.
4. The first telegraphic messages crossed the Atlantic ocean in 1858 by a cable 3,000 km long that was laid between Newfoundland (in Canada) to Ireland. The conductor in this cable consisted of seven copper wires, each of diameter $d \approx 0.73 \text{ mm}$, bundled together into a single conductor and then surrounded by an insulating sheath to protect it from seawater.

- (a) Using a resistivity for copper of $\rho_{\text{Cu}} \approx 3 \times 10^{-8} \Omega \cdot \text{m}$, estimate to the nearest power of ten the resistance R of this first trans-Atlantic cable.
- (b) The return path of the current was the ocean itself. Assume that the return path consisted of a spherical metal electrode of radius $a = 10 \text{ cm}$ stuck into the ocean near Newfoundland's shore, and a separate identical electrode stuck into the water near Ireland's shore. Estimate to the nearest power of ten the resistance between these two electrodes: is the resistance of the seawater across the Atlantic ocean between the two electrodes bigger or smaller than the resistance of the copper cable?

Some suggestions: assume that the current flows from one electrode to the other along some approximately cylindrical region in the ocean whose cross-sectional width really doesn't matter (if one is just trying to find out which resistance is bigger, can you see why a precise value for the width of the current region doesn't matter?) but you can take to be of order a kilometer in diameter, and use the resistivity for seawater of the previous problem, $\rho \approx 0.25 \Omega \cdot \text{m}$. This cylindrical region is in series with the resistances of the electrodes themselves, which you will see in lecture to be approximately $\rho/(4\pi a)$ for a highly conducting metal sphere of radius a immersed in a large medium of resistivity ρ . (The expression $\rho/(4\pi a)$ has the correct physical units for resistance given the general formula $R = \rho(L/A)$.)

5. Over modest changes in temperature T , the resistivity of most substances increases approximately linearly with temperature according to the relation

$$\rho(T) = \rho_0 [1 + \alpha (T - T_0)], \quad (2)$$

where ρ_0 is the resistivity at some reference temperature (often taken to be room temperature $T_0 = 20^\circ \text{C}$ in various tables such as Table 30.2 on page 879 of the Knight 3e book) and where the coefficient α is known as the **temperature coefficient of resistivity** and varies with the choice of conductor. It is often a serious nuisance that resistances change with temperature, e.g., if you need to fly some instrumentation up in space, electronic properties will change as various components become colder, and so scientists and engineers will often try to design temperature-compensated circuits whose properties are insensitive to changes in temperature over some range.

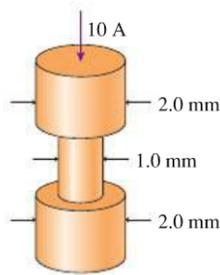
- (a) Show that two wires with different temperature coefficients of resistivity can be combined in series to produce a wire whose resistance does not change with temperature over moderate temperature changes. More precisely, consider a wire with cross-sectional area A , length L_1 , resistivity ρ_1 , and temperature coefficient α_1 that is connected end-to-end with a second wire whose corresponding parameters are A , L_2 , ρ_2 , and α_2 . Then show that if the condition

$$\rho_1 L_1 \alpha_1 + \rho_2 L_2 \alpha_2 = 0 \quad (3)$$

holds, then the total resistance of the two wires is independent of temperature for small variations in temperature.

- (b) But is this practical? If one wire is made of copper ($\rho_1 \approx 1.7 \times 10^{-8} \Omega \cdot \text{m}$, $\alpha_1 \approx 3.9 \times 10^{-3} \text{K}^{-1}$) and the other of graphite (carbon has $\rho_2 \approx 3500 \times 10^{-8} \Omega \cdot \text{m}$, $\alpha_2 \approx -0.5 \times 10^{-3} \text{K}^{-1}$), what is the ratio $L_{\text{Cu}}/L_{\text{graphite}}$ of the lengths of the two wires to achieve a temperature-compensated wire?

6. An aluminum wire consists of three 2.0 mm-long cylindrical segments with dimensions as shown in this figure:



The current entering the top segment is $I = 10$ A. For each of these three segments, find these six quantities to two significant digits: (a) current I ; (b) current density J ; (c) electric field E ; (d) drift speed v_d ; (e) electron current i ; and (f) resistance R . (Note: the resistivity of Al is $\rho \approx 2.8 \times 10^{-8} \Omega \cdot \text{m}$.) You should summarize your answers in a table with three columns for an easy comparison of the values.

7. If the electric field inside a metal conductor becomes so large that the drift speed v_d of the electrons becomes comparable in magnitude to the speed of the electrons themselves, Ohm's law fails and the current I is no longer simply proportional to the voltage difference V across the ends of the wire. To explore how Ohm's law might change in this circumstance, assume that you have a straight metal wire of length L , cross-sectional area A , electron density n_e (number of electrons per meter cubed), and such that a voltage difference V is imposed across the ends of the wire. Assume further that the collision time τ for an electron to collide and change the direction of its motion to some random new direction is no longer a constant but varies with the electric field magnitude E according to the expression

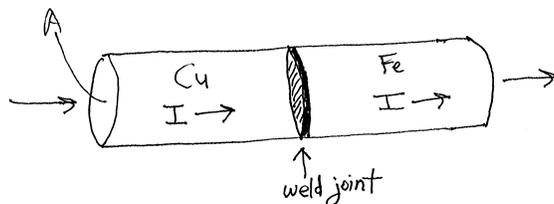
$$\tau = \tau_0(1 + \alpha E), \quad (4)$$

where $\tau_0 > 0$ and α are constants. (You can think of Eq. (4) as the first two terms of a Taylor series of $\tau(E)$ about $E = 0$, so a good approximation for sufficiently small E .) Ohm's law corresponds to the case $\alpha = 0$.

- (a) Discuss whether the coefficient α should be positive or negative.
- (b) Deduce and give a mathematical expression that shows how the current $I(V)$ flowing through the wire now depends on V . (Note that your expression should reduce to Ohm's law in the limit $\alpha \rightarrow 0$.) Sketch qualitatively the $I - V$ curve of current $I(V)$ versus potential difference V .
8. Consider two conducting rods of equal length L , each made of a substance with resistivity ρ . One rod is a cylinder of radius a , while the other rod is a truncated cone that tapers (or widens) linearly from a radius a at one end to a different radius $b \neq a$ at the other end. Show that the end-to-end electrical resistance R of the conical rod is a/b times the end-to-end resistance of the cylindrical rod.
- A hint: the cone can be thought of as the limit of a stack of thin disk-like cylinders of varying radius, each of small thickness dx , so use integration to add up the resistances of the stack of disks. For this problem, also assume that the current density J is uniform over and perpendicular to each infinitesimal slab (which is not such a great approximation in general).

9. The space between two coaxial metallic cylindrical shells of length L and inner radius R_1 and outer radius R_2 is filled with a non-metallic resistive medium with resistivity ρ .
- (a) What is the resistance R between the inner and outer cylinders in terms of L , R_1 , R_2 , and ρ ?
- (b) To three significant digits, what is the current I in amperes that flows between the two shells for $\rho = 30.0 \Omega \cdot \text{m}$, $R_1 = 1.50$ cm, $R_2 = 2.50$ cm, and $L = 50.0$ cm, when a battery with $\mathcal{E} = 10.0$ V is connected across the two shells?

10. A straight cylindrical piece of copper wire with cross-sectional area $A = (2 \text{ mm})^2$ is welded to a straight cylindrical piece of iron wire of the same diameter to form a long straight wire, of which a segment of length $2L$ is shown here:

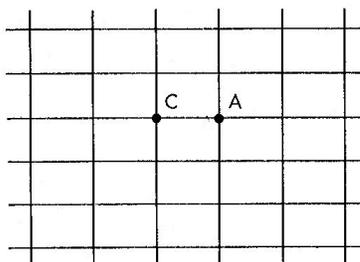


A current I flows through the copper wire and through the iron wire as shown. The resistivities of copper and iron are respectively $\rho_{\text{Cu}} \approx 2 \times 10^{-8} \Omega \cdot \text{m}$ and $\rho_{\text{Fe}} \approx 1 \times 10^{-7} \Omega \cdot \text{m}$.

- By applying Gauss's law to a small cylinder that is perpendicular to the weld and has ends in the Cu and Fe wires, explain why a net electrical charge will accumulate at the Cu-Fe boundary when a current is present. Also explain what is the sign of the accumulated charge for this particular wire and current.
 - Estimate to the nearest power of ten the number of charges (that is, some integer multiple of the fundamental unit of charge e) that will accumulate when a current of $I = 10 \text{ A}$ flows through this wire. Discuss briefly whether this charge would be easy to measure experimentally.
11. **Time to Complete This Assignment**
- To one significant digit, please give the time in hours that it took you to complete this homework assignment.

12. **Optional Challenge Problems**

- Challenge problem 6.1: A planar rectangular wire mesh of infinite extent



has 1 A of current fed into it at point A and 1 A of current taken from it at point C. What is the current through the wire AC?

- Challenge problem 6.2: A conductor is charged by repeated contacts with a metal plate which, after each contact, is recharged to have charge Q . If q is the charge of the conductor after the first contact, what is the ultimate charge on the conductor?
- Challenge problem 6.3: By what percentage does the capacitance of a spherical capacitor change when its surface is dented with a hammer such that its volume decreases by three percent?