Physics 162: Answers to End-of-Class Questions

February 6, 2015

If physicists cannot effectively calculate the mass of a proton, how did they come up with the mass of 10^{-27} kg that we use in our calculations?

The value you mention $m_p \approx 1.67262128 \times 10^{-27}$ is an experimental measurement of the mass of a proton, and was obtained by shooting a beam of protons with a known speed into a known magnetic field, measuring the radius of the resulting curved circular beam, and then using the known charge of an electron to deduce m_p. (You will be learning all these details in Physics 162 in about three weeks.)

What I mentioned in class were attempts by physicists to calculate the mass of a proton theoretically and mathematically from first principles, just knowing a few basic experimental facts like the charge of an electron, the three-dimensionality of space, and the experimental strength of the weak and strong interactions (roughly the equivalent of knowing K and G for these other interactions).

Physicists have developed a powerful mathematical theory called "The Standard Model" that unifies and explains an enormous amount of experimental knowledge about fundamental particles and how they interact via the electrical, weak, and strong interactions. Theorists believe that some fundamental experimental values like the mass of a proton or the mass of a neutron can be deduced from this theory but the calculations are extremely difficult because the inside of a proton or neutron are actually complicated time-varying many-particle systems involving three quarks and so-called gluons, and the number of particles is not constant.

We discussed today that the so-called electrostatic self-energy of a homogeneously charged proton $(3/5)Ke^2/d$ is about 0.1% of the experimental mass of a proton, and it turns out that the mass of the three quarks in a proton or neutron only contribute about one percent more of the total mass. The rest of the proton mass arises from the energy of interactions of the quarks and gluons (via $E = mc^2$) and adding up this energy is what is so difficult mathematically. Massively parallel powerful computers using a mathematical theory called lattice gauge theory and running for months have finally started giving proton masses with better than one percent accuracy compared to the experimental value, but there is still much that is not well understood.

Do you think that only geniuses can be successful mathematicians or physicists?

Definitely definitely not. The vast majority of valuable significant science and mathematics research is done by people who are not certified geniuses, and even many people who win Nobel prizes are not geniuses in any familiar sense of the word. A lot of superb science is also done by people who were not particularly impressive academically in college or in graduate school, you don't have to be an A student in math, physics, etc to be a successful or even exceptional scientist.

A key to success in mathematics, physics, and many other fields is having a strong passion and curiosity, there is something that interests you and you really really want to know the answer. You do need a certain minimal fluency with key foundational math and physics, but you don't have to be exceptionally good in an academic sense.

There are a few branches of physics where certain skills are crucial for the branches to advance, and perhaps you start running into people that would be called geniuses. So string theory, which concerns attempting to unify gravity with quantum mechanics, turns out to involve particularly advanced and pure mathematical styles of reasoning and so that is a branch of physics that requires a much stronger mathematics knowledge and skill than other branches. But even here, I would say that only a small fraction of string theorists would be called geniuses. And similarly there are experimental problems that require unusually creative or difficult ways to make progress, and you run into a few genius-like experimentalists who seem to have astonishing abilities to come up with a novel way to measure something and make the experiment work.

Where genius seems to play a key role are the rare times in a scientific field when everyone is stuck, say figuring out how to unify gravity with quantum mechanics, or trying to understand how brains work, or trying to understand fluid turbulence, or trying to understand the origin of life, or trying to create artificial intelligences, and then some person shows up who has a really different way of thinking about things and that person's insights cause everyone to change what they are doing. These people are often not geniuses in the prodigy sense, often they are not spectacular with mathematics or even with physics, they were not doing calculus or writing symphonies at age 10. What makes them special is that they are thinking in different ways than the rest of the scientific community.

But modern science and technology would come to a grinding halt if society had to rely on the few bona fide geniuses to make progress. There are many many interesting open scientific questions and most of you are perfectly capable of making major contributions with just a B level course knowledge of math and physics, provided that you are interested enough to work and think hard about some problem that interests you.

Why is it that there can never be an actual vacuum?

It has to do with the time-energy uncertainty principle of quantum mechanics, $\Delta t\,\Delta E > \hbar$. This states roughly that, over time intervals of duration Δt, it is not possible to measure or know the energy of a system to an accuracy better than ΔE. (So I am afraid that energy is not really conserved after all, since it is not possible to measure the energy accurately over very short time intervals.) For really tiny time intervals, the uncertainty ΔE in the energy can be large enough that the corresponding mass equivalent, $\Delta E/c^2$ (via $E = mc^2$) can exceed the mass of particles like electrons or positrons. So it turns out that, over sufficiently short time intervals, particles can spontaneously appear and disappear, representing allowed variations in energy over short time intervals. The appearance and disappearance of these so-called virtual particles have distinct measurable effects such as shifting the energy levels of hydrogen (Lamb effect) or causing forces to appear between two parallel neutral plates in vacuum (the Casimir affect).

So it is impossible for a true vacuum to exist, over sufficiently small time intervals, there are particles bursting into existence and then disappearing rapidly even in empty space, and this has been confirmed experimentally.

One reason why string theory is so difficult is that, on extremely short time intervals (of order the Planck time 10^{-43} s), one can "borrow" so much energy that the resulting mass collapses to a black hole, that is one can have virtual tiny black holes bursting into existence and disappearing over such short time intervals. Since black holes cause huge distortions of space and time near their surface, it becomes difficult to talk know what one means by time and space over such brief time scales.

Are there any human organs that use more than 7 W of power?

Actually, an adult human brain consumes about 15 W of power, with half of that power being associated with pushing charged ions across the 70 mV potential difference across the neuronal cell membrane, the rest of the power going into metabolic synthesis of proteins that are essential for brain function.

The other power-intensive organ is of course your heart, which (if I remember correctly) turns out to be comparable to the human brain in terms of power, again about 15 W. The heart is rather ineffecient though, it consumes 15 W but produces only about 2 W of actual mechanical pumping power that pushes blood around the body.

If you ground an outer conducting shell when the inner shell has a charge of $-Q$, how does the outer shell resist losing charge?

Assume we have two concentric conducting shells of radius R_1 and radius $R_2 > R_1$, with charges $Q_1 > 0$ and charge $Q_2 = -Q_1 < 0$ respectively. Next, assume we touch a ground wire, which is some wire whose other end is connected to a long metal rod that is driven into the ground, to the outer shell. What do we expect to happen?

This question gets to the essence of what electric potential implies: differences in electrical potential between two locations correspond to a change in electrical potential energy and therefore causes mobile charges to physically move between the two locations, converting the electrical potential energy to kinetic energy exactly in the same way that releasing a ball above the surface of the Earth causes the ball to fall spontaneously to the Earth's surface. (The ball a height h above the Earth's surface has potential energy mgh and this gets converted into kinetic energy $(1/2)mv^2 = mgh$.) So differences in electric potential causes particles to move, with positive charges moving from high to low potential energy regions, while negative charges move in the opposite direction.

So your question becomes: what is the potential difference ΔV between the outer shell and the Earth? If this difference is zero, no charges will move along the wire. If the difference is non-zero, charges will move through the wire, either from the shell into the Earth or from the Earth onto the shell, until the potential difference becomes zero.

To calculate the potential difference between the outer shell and the Earth, think about the fact that the Earth acts like a really large conductor and so charges spread out over its entire huge surface until they are quite far apart. Thus the electric potential of the Earth is the same as the electric potential of charges that are effectively infinitely far away from each other, that is $V_{\text{earth}} = 0$ if we make the common assumption that $V = 0$ for charges infinitely far away from one another. Alternatively, we can choose as our electric potential reference point P a point at infinity (a point so far from everything of interest that its electrical interaction is negligibly small) and assume that $V(P) = 0$. Then because the charges on Earth are so spread out and far from one another, we expect $V_{\text{Earth}} \approx V_P = 0$.

With this subtle insight, we now realize that calculating the electric potential difference $V_{\text{shell}} - V_{\text{Earth}}$ between the outer shell and the Earth is the same physically as calculating the electric potential difference between the outer shell and some point very far away from the shells, a point at "infinity". This potential difference is

$$\Delta V = -\int_{\infty}^{R_2} E_{\text{ext}}(r)dr, \tag{1}$$

where $E_{\text{ext}}(r)$ is the electric field magnitude external to the outer sphere. Using a Gaussian spherical bubble of radius $R > R_2$ centered on the two shells, you should be able to show that $\mathbf{E}_{\text{ext}} = \mathbf{0}$ everywhere outside the outer shell and so the electric potential $V(R_2)$ of the outer shell with respect to infinity or with respect to the Earth is zero. So there is no potential energy difference between the outer shell and the Earth and so any charge on the outer shell will not move off the shell.

Let's consider one more situation, what if the outer shell had charge $Q_2 \neq -Q_1$ so the two shells together have a total charge $Q_1 + Q_2 \neq 0$. Now there is an electric field everywhere outside the outer shell with a value given by Gauss's law: $\mathbf{E} = K(Q_1 + Q_2)/r^2$, and the potential of the outer surface is now

$$\Delta V = -\int_{\infty}^{R_2} E_{\text{ext}}(r)\, dr \tag{2}$$

$$= -\int_{\infty}^{R_2} \frac{K(Q_1 + Q_2)}{r^2}\, dr \tag{3}$$

$$= \frac{K\,(Q_1 + Q_2)}{R_2}. \tag{4}$$

There is now a non-zero potential difference and charge will move on or off the outer shell ("falling" down the electric potential energy difference) until that potential difference becomes zero. So Q_2 will spontaneously change its value (with charge moving into the Earth or from the Earth onto the outer shell) until Q_2 has a new value Q_2' such that $V(R_2) = K(Q_1 + Q_2')/R_2 = 0$ which implies $Q_2' = -Q_1$. So grounding the outer shell of a non-neutral system of shells does not remove all the charge on the outer shell, but just enough to cause the entire system to become neutral.

A challenge: what if the same two shells with charges $\pm Q$ are placed side by side, rather than one inside the other, so that there is a large distance d between their centers ($d \gg R_1, R_2$) which allows you to ignore polarization effects of one shell on the other shell. If you now touch a ground wire to shell 2 with charge $-Q$, what will be the final charge Q_2' on that grounded shell?