Physics 162: Answers to End-of-Class Questions

January 21, 2015

Should we know how to derive all of the electric field equations or know the geometry of the problems and use the equations as they are?

You should know how to calculate electric fields of single objects, or of groups of objects, at the level of the Knight Chapter 26 and of your second homework assignment. So you should be able to derive the electric field on the axis of a charged ring, you should be able to calculate the electric field on the bisector of a line segment or of an infinite line, and you should be able to use superposition and integration to calculate the electric fields of new objects that are combinations of familar objects, e.g., of a infinite planar strip of charged material or on the axis of a charged cone, or of a uniformly charged sphere with a spherical void of given location and size.

In quizzes and exams, I will provide you with formulas for the electric fields of lines or of disks or I will provide you with the value of any integrals that are not quick to work out (I would say that the integrals of x^m, $\sin(cx)$, and $e^{cx}$are quick and easy, but not of $1/(x^2+a^2)^{3/2}$), but you will still need to know how to obtain an integral corresponding to the answer and how to manipulate the integral into a form that corresponds to a given evaluated integral.) I will be showing you this in today's (Thursday's) class and you will have a chance to master these ideas through the current homework assignment.

I would like to know more about the physical reasons as to why a sharp tip produces an electron cloud.

We will discuss this in class when we get to Chapter 29, e.g. see Section 29.4 of the textbook, especially Figure 29.19. A key insight is that a charged conducting object has a surface charge density σ that is not uniform over the surface of the object, with σ being large where the local radius of curvature of the surface is small (near sharp points or bumps) and being small where the local radius of curvature is large (flat regions). The tip of a cone corresponds to a tiny radius of curvature and so the surface charge density σ can be quite large, which makes the electric field quite large and which can ionize nearby atoms.

How do the electric fields of the ends of lines and planes work?

This is a bit tricky to answer because you have to ask whether you are talking about idealized mathematical non-physical models of charged lines and charged planes or talking about actual lines and planes made up of physical discrete atoms. For the former, the electric field can diverge (become infinite in magnitude), for the latter the electric field does not diverge.

Let's work out a concrete example: consider an idealized charged rod that we model as a line segment of length L and uniform positive charge density $\lambda > 0$. Let's place the rod on an x axis with one end at location $x = 0$ and the other end at location $x = L$, and let's consider a point $P = (d, 0)$ on the x-axis at a location $d > L$ to the right of the right side of the line segment. Let's then calculate what is the electric field $\mathbf{E} = E_x(d)\hat{\mathbf{x}}$ at point P created by the entire line segment and explore what happens as the point P approaches the right side of the rod, i.e., what happens to E_x as $d \to L$.

We can mentally break the rod up into little infinitesimal equal pieces of length dx and charge $dq = \lambda dx$. If a given infinitesimal piece has coordinate x, with $0 \le x \le L$, then because it is an infinitesimal piece we can treat it as a point object and calculate its electric field dE_x at P using Coulomb's law:

$$dE_x 1 = \frac{K dq}{(d-x)^2} = \frac{K(\lambda dx)}{(d-x)^2}. \tag{1}$$

We can then use integration to get the total electric field at P by adding up the contribution of each little piece of the rod:

$$E_x(d) = \int_0^L \frac{K\lambda dx}{(d-x)^2} = K\lambda \left[\frac{1}{u}\right]_d^{d-L} = K\lambda \left[\frac{1}{d-L} - \frac{1}{d}\right]. \tag{2}$$

Note that this expression makes sense. For example, for $d > L$, it is positive since $1/(d-L) > 1/d$ since $d - L < d$, and E_x has to be positive since electric fields point away from positive objects. Also note how if P is far away from the rod, $d \gg L$, we can use the generalized binomial theorem to write

$$\begin{aligned}
E_x(d) &= K\lambda \left[\frac{1}{d-L} - \frac{1}{d}\right] && (3)\\
&= \frac{K\lambda}{d}\left[\frac{1}{1-L/d} - 1\right] && (4)\\
&= \frac{K\lambda}{d}\left[\left(1-\frac{L}{d}\right)^{-1} - 1\right] && (5)\\
&\approx \frac{K\lambda}{d}\left[1 + \frac{-1}{1}\cdot\left(\frac{-L}{d}\right) - 1\right] && (6)\\
&\approx \frac{K(L\lambda)}{d^2}, && (7)
\end{aligned}$$

and this last expression is the electric field of a point charge of charge $L\lambda$, which is the total charge of the rod. So far from the rod, the rod acts like a single point charge, which also makes sense.

Now that we have some faith that Eq. (2) is scientifically reasonable, you can see that as $d \to L$ in Eq. (2), the electric field E_x diverges to infinity since $1/(d-L)$ approaches positive infinity, the electric field diverges at the ends of the rod. This would not happen for a point charge like an electron approaching an actual physical rod since, as the electron gets closer and closer to the rod it starts getting close to particular atoms in the rod and you can't get a divergence unless the electron physically approaches some other electron, which won't happen because of the quantum mechanical structure of atoms. This explains why I didn't blow up Room 150 when I charged up a rubber rod with cat fur and brought it close to (effectively touching) other charged objects.

From the equation for the electric field $E_z(z)$ on the perpendicular bisector of a rod of length L and charge density λ on page 759 of the Knight book (which we will derive in Thursday's class)

$$E_z(z) = \frac{K(\lambda L)}{z\sqrt{z^2 + (L/2)^2}}, \tag{8}$$

you can see that the electric field also diverges as one approaches the midpoint of the charged rod (that is, as $z \to 0$), which suggests the correct guess that the electric field diverges as one approaches any part of the rod, not just the end points. This divergence is an artifact of the rod being treated as a line segment with zero thickness, and goes away if we were using a cylindrical rod of finite thickness instead, as we will discuss in Chapter 27 on Gauss's law.

According to Knight and our discussion today in class, the electric field produced by a mathematical infinite plane is finite right up to the surface of the plane but it would diverge at the edges of a finite square or disk.

Do cells produce such a strong E field to also counteract the diffusion of ions naturally through their membranes? Is the E field related to a cell's Nernst potential?

It is the opposite of what you say: the strong electric field across the cell membrane enhances and speeds up the diffusion of charged particles across the lipid bilayer membrane, if and when some hole opens up across the membrane, say by some channel membrane opening up. Diffusion of neutral particles, caused by a concentration gradient, occurs at a certain rate but this rate is enhanced if there is some force like gravity or

an electric force that pushes the particles in the direction they are diffusing. (This is called "drift diffusion" if you want to look it up.)

The electric field magnitude E across the membrane is precisely related to the electric potential ΔV across the membrane by the relation $\Delta V = Ed$ where $d \approx 4\,\text{nm}$ is the thickness of the membrane. The value of the potential ΔV is indeed determined by various concentration gradients via Nernst's equation.

But we haven't discussed yet in Physics 162 what is an "electric potential ΔV" and this will be an important topic this semester. I do plan to discuss further the electric potential of cell membranes, e.g., when we discuss neurons and what determines how rapidly the potential can change, which is related to how fast your brain can process information.

Why are we able to represent a void as an opposing charge density?

A void is an empty space that has no charge. If you consider some charged object like a Styrofoam sphere that is uniformly filled everywhere with some volume charge density ρ and you create a void inside that object, you have removed the charge from that void.

You get exactly the same effect by superposition if you were to mathematically merge an object with the shape of the void but with charge density $-\rho$ in the location of the void inside the sphere. (It is hard to see how to do this physically but it can be done mathematically by just positioning the $-\rho$ object in the location of the void inside the original object.) The charge densities then add up to zero and so acts like a void of zero charge.

(Or as one 162 student nicely pointed out in his end-of-class question, you also get the same answer if you subtract the electric field from a small sphere with the same volume charge as the big sphere and with the same location as the void. This works since subtracting a sphere with density ρ is the same as adding a sphere to the desired location with density $-\rho$.)

To flesh out our discussion in class, let's consider a sphere of radius R and constant positive charge density ρ whose center is placed at the origin of an xyz-Cartesian coordinate system. Then I argued in class that, at any point P outside this sphere, the electric field at that point is that of a point charge at the center of the sphere with total charge $Q = \left[(4/3)\pi R^3\right]\rho$. (We will prove this rather trivially using Gauss's law next week.)

Consider a spherical void that we cut out of this sphere of radius R_1 and let's put the center of this void at coordinate $(x_1, 0, 0)$ and ask: what is the electric field $E_x(d)$ produced by this "sphere with void" at a point $P = (d, 0, 0)$ on the x axis where $d > R$ so the point lies outside the sphere? (This is a simpler case when the spherical void, the center of the sphere, and the point of interest all line up.) Then I claim that the electric field $E_x(P)$ at P is the sum of the electric field E_1 due to the original sphere without a void

$$E_1 = \frac{K\left(\left[(4/3)\pi R^3\right]\rho\right)}{d^2}, \tag{9}$$

plus the electric field due to a sphere of radius R_1 and charge density $-\rho$ at the location of the void:

$$E_2 = \frac{K\left(\left[(4/3)\pi R_1^3\right](-\rho)\right)}{(d - x_1)^2}, \tag{10}$$

since the sphere filling the void with density $-\rho$ itself acts like a point charge at location x_1 with total charge $\left[(4/3)\pi R_1^3\right](-\rho)$. So we can find that the total electric field at P is this:

$$E_x(d) = \frac{4\pi K\rho}{3}\left(\frac{R^3}{d^2} - \frac{R_1^3}{(d - x_1)^2}\right). \tag{11}$$

I encourage you to explore the more general case where the point of interest P lies at some general point $P = (x, y, z)$ outside the sphere that is not aligned with the center of the sphere and the center of the void so that

you need to use some vector arithmetic to calculate the electric field at P. But I hope you see that the math reduces to adding up the electric field at a point due to two point charges, one positive and one negative.

Does the phenomenon of generating light by a moving charge explain photon emission upon electron excitation?

These are two separate mechanisms: an accelerating charge generates light in a way that can be explained without quantum mechanics, using Maxwell's equations that we will discuss towards the end of the semester. And an atom can become excited (through a collision) and then emit a photon (light particle) upon becoming 'de-excited" and this is a quantum mechanical process that has no explanation via Maxwell's equation, since the idea of photons and quantized states is incompatible with the "classical" pre-quantum ideas of Maxwell's equations.

One can explain radiation of light by accelerating charges within quantum mechanics but it is a graduate-level topic.

Why are infinite sheets or lines a good approximation of real world objects?

This is explained in the Knight book if you read it carefully. It is only under certain circumstances that it is a good approximation to calculate the electric field of a charged rod or charged rectangle as if they were infinitely extended objects. The circumstances are when the point P of interest (where you want to know the electric field) is much closer to the object (line, plane) than the physical size of the object itself.

For example, consider a charged cylindrical rod of length L, radius R, and volume charge density ρ. (This is a better model of a rod than a line segment for which $R = 0$.) Then for a point P far away from the rod, so the distance d of P to the center of the rod satisfies $d \gg R$, $d \gg L$, the rod is best approximated as a point charge of total charge $Q = V\rho = \left[(\pi R^2)L\right]\rho$, and you don't want to think of the rod as infinitely long.

But if you choose a point P that is close to the midpoint of the rod, so the distance d of P to the surface of the rod satisfies $d \ll L$, then it is a good approximation to treat the rod as having infinite extent.

Similarly, if you have a charged flat square of plastic with side L and uniform surface charge density σ, then for points P far from the square, the electric field is that generated by a point charge of total charge $Q = A\sigma = L^2\sigma$. But if the point P is close to the center of the square and its distance to the square d satisfies $d \ll L$, then it is accurate to treat the square as an infinite plane.

In the case of an irregular surface that cannot be divided into rings or other curves, does the process change at all or is normal integration still the best route?

With modern computers and algorithms, it is pretty much trivial to calculate the electric field of any arbitrary arrangement of point charges and charged objects of any shape. All you need to do is break up the objects into sufficiently small regions that the charge density ρ over that region is approximately constant and such that you can treat the regions as approximately point particles, and then you can use Coulomb's law to get the electric field at any point $P = (x, y, z)$ of interest by adding up all the electric field vectors at P generated by the charges on each of the tiny cubes.

So the skills we are learning just now in Chapter 26 are good for getting some intuition about how electric fields arise from some simple symmetric objects, but are rather useless for calculating analytically the electric fields generated by realistic objects that are not simple cylinders or spheres or planes or unions of those objects. But you do know now how to write a computer code that will calculate the electric field of any general charge distribution.

In more detail, consider some object of arbitrary shape such that the charge density at any point (x, y, z)

in the object is given by $\rho(x, y, z)$. (So someone has to give you ρ as a function of space.) Next consider partitioning the object into a union of identical tiny parallel cubes of volume V that are each centered on coordinates $\mathbf{x}_i = (x_i, y_i, z_i)$ that one can label from $i = 1$ to $i = N$ where N is some large integer representing the total number of tiny cubes needed to cover the object. The cubes are so tiny that the charge density is approximately constant over their volume, so the charge of a given cube is simply $Q_i = \rho(\mathbf{x}_i)V$. We also assume the cubes are so tiny that they can be treated as point charges.

Then a computer program would add up the following sum to approximate the electric field $\mathbf{E}(\mathbf{r})$ at some general point $\mathbf{r} = (x, y, z)$ in space:

$$\mathbf{E}(x, y, z) = \sum_{i=1}^{N} \frac{K(V\rho_i)}{(\mathbf{r} - \mathbf{r}_i)^2} \cdot \frac{\mathbf{r} - \mathbf{r_i}}{|\mathbf{r} - \mathbf{r}_i|}. \tag{12}$$

The approximation can be improved by using ever small boxes or by using a more clever algorithm to approximate the sum.

In Physics 174, we spoke about using technology similar to the method bats use to locate objects with ultrasound. Are there any similar techniques employing the method eels use to locate objects using electric fields?

The answer is yes. The method is called "electrical impedance tomography" (see the Wikipedia article by that name) and was indirectly motivated by thinking about how electric fish sense their environment.

Electric fish sense the world external to their body using electric fields they generate and then sense with skin sensors, while electrical impedance tomography tries to determine the internal structure inside of a body through surface electrical measurements.

Your question suggests an interesting experiment which I don't think has ever been done, which is whether an electric fish can sense its own internal body structure via its electrical sensors. It would not be hard to try this experiment, e.g. by implanting neurons with electrodes in the part of the brain that processes the electric field sensors and then moving some object around inside the fish.

If the electric field physically exists, does it mean that it has energy itself?

Good thinking on your part: the answer is yes. It takes energy to create electric fields and one can extract energy from electric fields so indeed electric fields can be thought of as having energy.

Later this semester (see page 855 of the Knight book), we will derive the fact that the energy per unit volume stored in an electric field is given by

$$u = \frac{\epsilon_0}{2} E^2, \tag{13}$$

where ϵ_0 is the vacuum permittivity and is basically Coulomb's constant written upside down: $\epsilon_0 = 1/(4\pi K)$. So if you know the electric field $\mathbf{E}$ over some region of space, over some tiny region of volume V, the energy due to the electric field in that region is $(\epsilon_0/2)E^2 \times V$.

When you feel the warmth of sunlight on your face, you are feeling the energy of the electric and magnetic fields of light being transferred to your skin.

Can you calculate charge densities in dimensions greater than three? Are there applications of this?

Scientific and engineering problems exist in three spatial dimensions so any actual experimental problem would involve charge densities in one, two, or three spatial dimensions. (Since everything is made of three

dimensional atoms, technically you only need a three-dimensional charge density ρ to describe any charge distribution in nature, but it is helpful to consider linear or areal densities for idealized lines and areas.)

But theoretical physicists are not constrained by experimental reality and often consider situations that generalize the world as we know it. So string theorists hypothesize that there might be nine other spatial dimensions and then one could consider charge densities in dimensions up to nine.

Perhaps even stranger and neater, scientists and mathematicians have known for nearly a hundred years of sets and naturally occurring objects called 'fractals' that have spatial dimensions that are non-integer. For example, the clouds you see in the sky are highly wrinkled objects that technically have infinite surface area and zero volume, with a spatial dimension of $D \approx 2.5$, intermediate between a surface and volume. (This is one reason why weather simulations have a hard time predicting the weather accurately, it is hard to calculate accurately the reflection of light off a fractal cloud.) The lungs in your body are another fractal system with a dimension between 2 and 3.

So physicists (and other scientists and engineers) have considered different examples of fractal distributions of charge because it allows them to consider certain interesting possibilities that can actually occur in nature. As one example relevant to 162, the path traced out by lightning and by intense sparks are fractals, see the Wikipedia article "Lichtenberg figure" and this pretty figure of an electrical discharge through a block of plastic:

`http://en.wikipedia.org/wiki/Lichtenberg_figure#mediaviewer/File:PlanePair2.jpg`

If our sensory organs have reached maximum responsivity, is there no more room for evolution unless the environment changes?

I would agree with this statement.

For vision, audition, and olfaction, evolution seems to have produced sensors that operate right at the limits of sensitivity allowed by the laws of physics, given a few additional facts such as that the sensoring devices are working at room temperature (which means the sensors are working in the presence of an unavoidable thermal noise of a certain magnitude) and are made of certain kinds of biomolecules.

Scientists can improve the performance of sensors by doing things like operating them at much lower temperatures (reducing thermal noise) or by making them out of materials that no organism has learned how to make such as certain metal or semiconductor materials (e.g., look up a device called a SQUID, which is capable of measuring the smallest possible magnetic fields).

If our environment changed, e.g., if the average temperature of the world were to increase substantially (looks like a possibility) or if the color of the Sun were to change (which is going to happen, the Sun will become much more red colored as it enters a giant red star phase in several billion years), then sensors like our eyes will likely change by evolution to function better in the new environment.

I am confused about charged objects and objects that polarize. Why wouldn't the Van der Graaf generator have concentrated electrons on the outside, facing the object like this (drawing provided)?

If one charges up a Van der Graaf generator, the dome becomes positively charged and the positive charge spreads out approximately uniformly over the metal dome. (More precisely, it is the low-mass electrons in the metal that are mobile in a metal, not the heavy atomic nuclei that are locked in place in a crystal lattice, and it is the deficit of electrons that spreads out evenly to make the dome uniformly positive in charge.) When an object is brought close to the dome such as that small little ball connected to a grounding wire, there is indeed some rearrangement of charge on the dome corresponding to the dome becoming polarized. But how much the dome becomes polarized depends on how strong is the electric field of the dome (how big

is the total charge on the dome) and properties of the object being brought near the dome e.g., whether it is large or small.

While it is easy to calculate polarization precisely using computers and Maxwell's equations or measure polarization experimentally, it is too hard to calculate analytically except for a very small number of problems, especially at the level of Physics 162 for which we assume you have not learned the mathematics of partial differential equations. So I am afraid we will have to be qualitative and somewhat incomplete and vague in our discussions of anything involving polarization.

An optional homework problem in about two weeks will give you a chance to solve analytically one of the few problems that can be worked out completely (namely a point charge near a spherical metal charged conductor) and that solution will tell you everything you need to know about how the metal sphere polarizes. In particular, you will be able to calculate at what precise distance the repulsion of a positive point charge from a positive sphere switches to attraction because of polarization.

Can we sense electric fields? Why or why not?

Humans and most land animals cannot sense electric fields through specialized electric field sensors like those of electric fish, sharks, and some insects. Humans can sense electric fields indirectly and very crudely e.g., if you bring your arm near a charged Van der Graaf generator, the hairs on your arm will get polarized and you can feel the the resulting electric force vector acting on those hairs.

The only land animal that I know of that has electric sensors is the duck-billed platypus.

"Electroreception" as it is called seems to have evolved only in animals that live in places where vision is poor or not possible such as muddy rivers or dark caves. Evidently, mammalian vision is so effective that once you have good eyesight, you don't benefit from sensing electric fields.

If you want to learn more, see the Wikipedia article "Electroreception".

Can we alter the electric fields across cell membranes, and so possibly improve the health of some person?

The answer is yes, we do know how to alter the electric field across cell membranes but I don't know if anyone has used this ability to avoid disease or cure a disease.

One way to alter the electric field is by physics, say by inserting a very fine electrode into a single cell and then connecting the electrode to some circuitry that produces a tiny charge at the tip of the electrode. Using this method, a scientist can dial up pretty much any desired electric field across the cell membrane, and scientists have then used this technique to make many important discoveries about how neurons and heart cells work. But this method can control only a few cells at most at any given time, which is often not enough to affect the health of a person except in cases where a few cells are the source of trouble, e.g., a focal center in the brain that causes a localized epilepsy, or the sino-atrial node of a person's heart that might not be functioning correctly.

Another, more modern way, to alter the electric field of cells is to use the full power of genetics. For certain species like flies and mice, biologists have developed extraordinary tools for creating animals with known precise changes to their genetics (so-called transgenic animals), and so biologists can change the properties of the proteins that are embedded in a cell's membrane and that determine the electric fields across the membrane.

In other cases, it is possible to use genetics to make an animal like a mouse grow certain proteins in its cells (say neurons) that don't normally exist in mammals, e.g., one can genetically insert genes from a certain algae that, when expressed in a mammalian brain, cause the electric field in the neuronal membranes to become sensitive to flashes of light, so you can control the electric field at will with light pulses. (See the

Wikipedia article “Optogenetics” to learn more about this revolutionary technique, which is likel to earn a Nobel Prize within the next few years.)