Print your name clearly: ______________________________

Signature: ______________________________

"I agree to neither give nor receive aid during this exam."

Physics 162 Midterm Exam

Professor Greenside
Wednesday, February 25, 2015

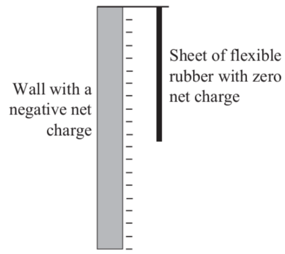
This exam is closed book and no electronic devices are allowed. The test will last the entire time from 10:05 am to 11:20 am. Please note the following:

1. Please answer *all* questions on extra blank pages, including multiple choice and true-false. On the extra pages, please use plenty of space and paper and write clearly. You will lose points if I can not *easily* read and understand what you write.
2. Unless otherwise stated, you need to justify your answers, at least briefly, including multiple choice and true-false questions.
3. Please write your name and the problem number at the top of each extra page. Sort your pages in order of increasing problem number before handing in your exam.
4. Don't hesitate to ask me for help during the exam if you don't understand some question.

Breadth

1. **(3 points)** True or false and why: for a metal wire carrying a current in some circuit, there is no excess charge anywhere inside or on the wire.
2. **(3 points)** True or false and why: if a cylindrical hole is drilled along the axis of a solid cylindrical wire that is carrying a fixed current I, then the magnitude of the electric field inside the wire will increase.
3. **(3 points)** True or false and why: the surface of a metal conductor in electrostatic equilibrium is not an equipotential surface if the conductor is being polarized by a nearby charge.
4. **(3 points)** True or false and why: if a non-conducting material with uniform volume charge density ρ is made into the shape of a pencil (cylinder with a conical tip), then the magnitude E of the electric field produced by this object is largest near the tip.
5. **(3 points)** True or false and why: consider a metal conductor in electrostatic equilibrium, and consider a point P on the surface of the conductor. Then the magnitude E of the electric field vector $\mathbf{E}$ at P and the surface charge density σ at P are related by $E = \sigma/(2\epsilon_0)$.

6. **(3 points)** (Note: no justification of your answer is needed for this problem.) A non-conducting wall is given a net negative charge. A neutral thin flexible sheet of rubber is then suspended from the ceiling near the charged wall as shown here:



The rubber sheet will

(a) not be affected by the charged wall since rubber is an insulator.

(b) not be affected by the charged wall because the rubber sheet has zero net charge.

(c) bend away from the wall due to the electrical repulsion between the electrons in the rubber and the negative charges on the wall.

(d) bend away from the wall due to polarization of the rubber molecules by the charged wall.

(e) bend towards the wall due to the polarization of the rubber molecules by the charged wall.

(f) do something else not described above.

7. **(3 points)** (Note: no justification of your answer is needed for this problem.) A battery is connected to a tungsten-filament light bulb so that the bulb is glowing. Which of the following statements is true about the electric field **E** inside the bulb filament?

(a) $\mathbf{E} = \mathbf{0}$ because tungsten is a metal.

(b) $\mathbf{E} = \mathbf{0}$ because a current is flowing.

(c) $\mathbf{E} = \mathbf{0}$ because any excess charges are on the surface of the filament.

(d) $\mathbf{E} = \mathbf{0}$ for reasons not given above.

(e) $\mathbf{E} \neq \mathbf{0}$ because the flowing current produces an electric field.

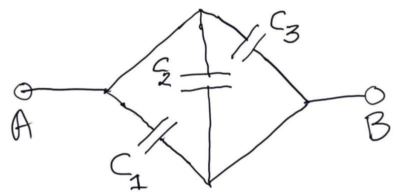
(f) $\mathbf{E} \neq \mathbf{0}$ because no current will flow without an applied electric field.

(g) $\mathbf{E} \neq \mathbf{0}$ for reasons not given above.

8. **(4 points)** The Earth acts like a large spherical conductor with radius $R \approx 6,400\,\text{km}$, with a negative surface number density of about 10^{10} electrons/m^2, that rotates once around its axis every 24 hours. To the nearest power of ten, the total current created by the rotation of the Earth's charged surface is

(a) 10^{-3} A (b) 10^{-1} A (c) 10^{1} A (d) 10^{3} A (e) 10^{5} A

9. **(4 points)** What is the effective capacitance between points A and B of this three-capacitor circuit with capacitances C_1, C_2, and C_3?



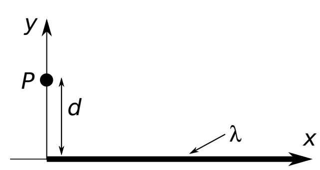
10. **(4 points)** A positive charge $Q > 0$ is transferred to a neutral solid metal cube. After electrostatic equilibrium is attained, the maximum electric potential V is located at

    (a) the center of the cube.
    (b) the edges of the cube.
    (c) the vertices of the cube.
    (d) the center of the faces of the cube.
    (e) any point on the cube, they all have the same potential.

11. **(4 points)** For a non-conducting cube that has a uniform charge density $\rho > 0$, the maximum electric potential V is located at

    (a) the center of the cube.
    (b) the centers of the edges of the cube.
    (c) the vertices of the cube.
    (d) the center of the faces of the cube.
    (e) any point on the cube, they all have the same potential
    (f) somewhere outside the cube.

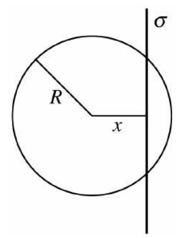
12. **(4 points)** Consider a semi-infinite uniformly charged wire with constant linear density $\lambda > 0$ that lies on the x-axis for $x \geq 0$, and consider a point P on the y-axis that lies a distance d above the end of the line as shown here:



Using superposition, but without using any calculus, determine the value of the y-component E_y of the electric field $\mathbf{E}$ at P in terms of d and λ.

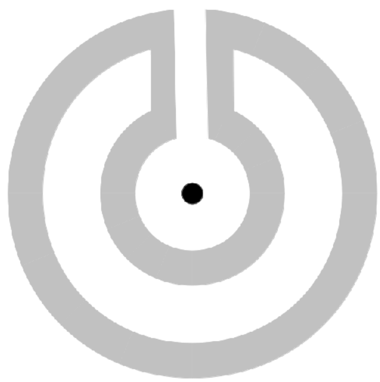
13. **(6 points)** You have a box of many identical 10 pF capacitors, each with a maximum voltage rating of 100 V. Explain how to create an effective capacitance that can be used in a circuit that requires a 20 pF capacitor with a maximum voltage rating of 200 V.

14. **(6 points)** An infinite charged plane with constant surface charge density σ intersects a spherical Gaussian bubble of radius R a distance x from its center as shown here:



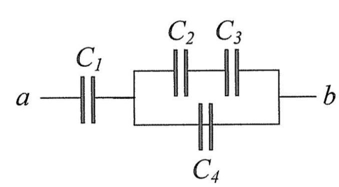
What is the electric flux Φ through the Gaussian bubble?

15. **(6 points)** A long neutral cylinder-like conductor has the following cross section:



where white areas denote vacuum and gray areas denote solid metal. If an infinite line of uniform positive charge density is placed along the axis of this shell as shown (the cross section of which is the black dot), **then indicate on this drawing** where the surface charge density σ is nonzero and what is its sign by drawing various + and − symbols on the figure. Explain your reasoning briefly.

16. **(8 points)** Given the following arrangement of four identical capacitors,



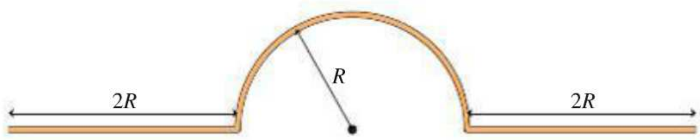
each with capacitance C, and given that a charge of magnitude $Q_3 = Q$ is observed on capacitor C_3 when a certain battery is connected to terminals a and b, what is the emf $\mathcal{E}$ of the battery in terms of Q and C?

17. **(8 points)** Nichrome is a non-magnetic alloy of nickel, chromium, and iron that is often used to make resistors. One end of a Nichrome wire of length $2L$ and cross-sectional area A is attached to the end of another Nichrome wire of length L and cross-sectional area $2A$. If the free end of the longer wire has an electric potential of 8 V and the free end of the shorter wire has an electric potential of 1 V, what is the approximate electric potential at the junction of the two wires?

    (a) 2.4 V (b) 3.3 V (c) 4.5 V (d) 5.7 V (e) 6.6 V

Depth

1. **(15 points)** A thin glass rod is heated and then bent into the following shape consisting of two line segments of length $2R$ and a semi-circle of radius R:



If this glass shape is now uniformly coated with a positive linear charge density $\lambda > 0$, what is the electric potential V at the black dot at the center of the half circle? (You need to evaluate any integrals that arise so that your final answer has no integrals in it.)

2. Consider a thick cylindrical tube made out of graphite that has length L, an outer radius R_2, and an inner radius $R_1 < R_2$. The resistivity of the graphite is ρ.

   (a) **(14 points)** The inner cylindrical surface and outer cylindrical surface of this graphite tube are coated with gold so that these become equipotential surfaces, and the positive and negative terminals of a battery with emf $\mathcal{E}$ are connected respectively to the inner and outer gold surfaces, in which case a current flows radially from the inner surface to the outer surface. For each of the following quantities, explain briefly whether their value decreases, stays the same, or increases as one progresses from the inner radius to the outer radius:

   i. electron density n_e.
   ii. current I.
   iii. current density J.
   iv. electric field strength E.
   v. electric potential V.
   vi. drift speed v_d.
   vii. collision time τ.

   (b) **(6 points)** For this same gold-coated graphite tube, deduce and give an expression for the resistance R_{radial} for a radial-flowing current between the inner and outer cylindrical surfaces in terms of ρ, L, R_1, and R_2. Then estimate this resistance in ohms to the nearest power of ten for the values

$$\rho \approx 3.5 \times 10^{-5}\,\Omega \cdot \text{m}, \qquad \text{L} = 0.8\,\text{m}, \qquad \text{R}_1 = 2\,\text{cm}, \qquad \text{R}_2 = 6\,\text{cm}. \tag{1}$$

   (c) **(6 points)** Consider now the case in which a current is made to flow longitudinally along the length of the same graphite tube, by first removing the gold on the inner and outer cylindrical surfaces and then creating gold contacts on just the two flat ends of the graphite tube. Deduce and give an expression for the resistance $R_{\text{longitudinal}}$ between the two flat ends, and estimate its value in ohms to the nearest power of ten using the same parameters in Eq. (1).

3. A certain multi-part capacitor (see figure on the next page) consists of:

   (a) two large parallel metal plates of area A that are separated by a distance $5d$;

   (b) a solid neutral conducting slab of thickness d and area A that is placed a distance d from and parallel to the left metal plate.

   (c) and a solid neutral dielectric slab of thickness d, area A, and dielectric constant $\kappa = 2$ that is placed a distance $3d$ from and parallel to the left plate.

   The figure on the next page shows cross-sections of the plates and slabs, which are all perpendicular to the plane of the paper. The left metal plate has a positive charge $Q > 0$ and the right metal plate has an opposite negative charge $-Q < 0$.

   (a) **(10 points)** On the two pairs of axes in the figure below, draw qualitatively the x-component $E_x(x)$ of the electric field, and draw qualitatively the electric potential $V(x)$ over the range $-d \le x \le 6d$, with the assumption that $V(0) = 0$. (Please note that this range extends beyond the capacitor plates!).

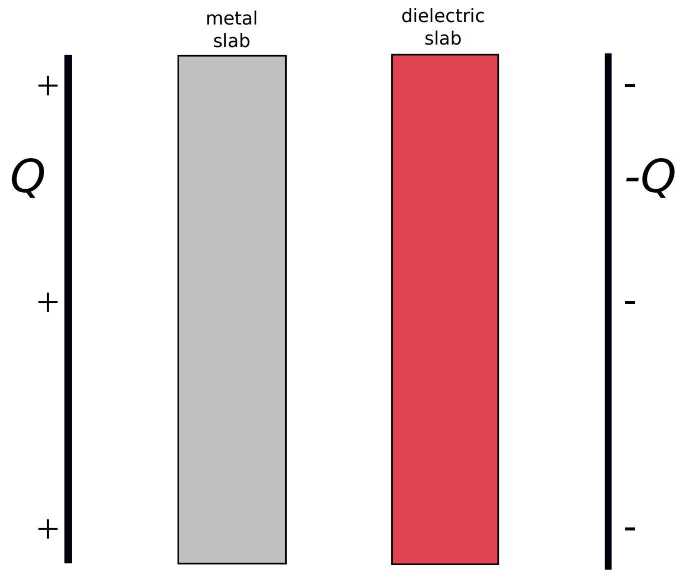
   (b) **(8 points)** What is the capacitance C of this object in terms of A, d, and κ?

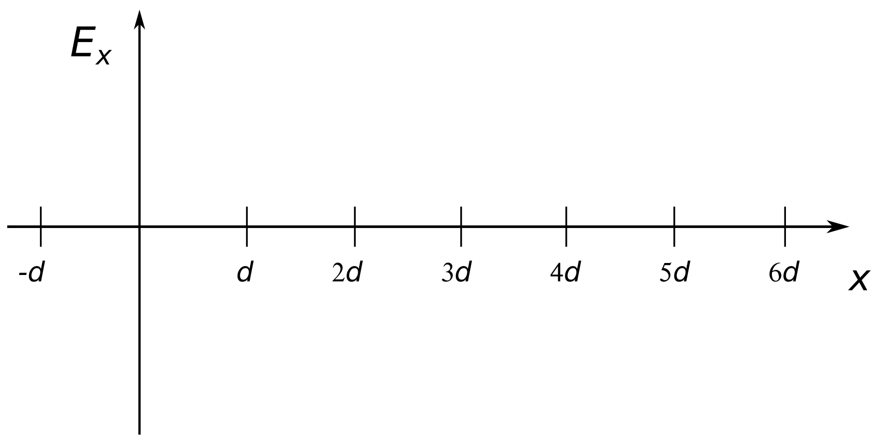
   (c) **(6 points)** For each of the following cases, explain briefly whether the energy stored in this capacitor increases, stays the same, or decreases:

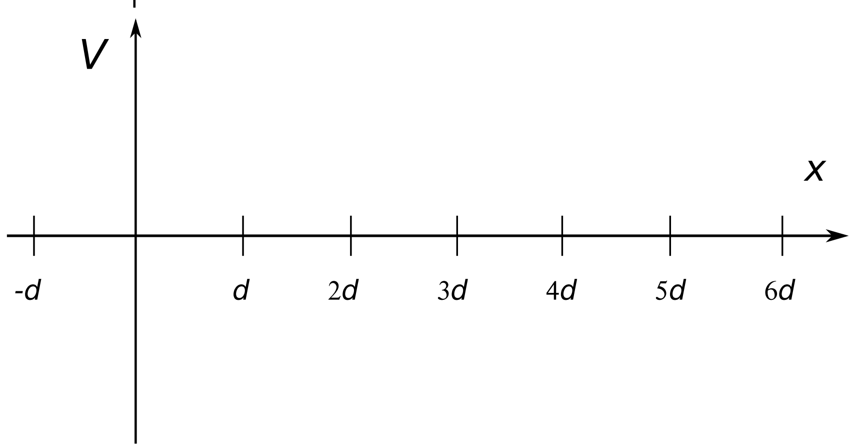
      i. The metal slab is removed from the capacitor with everything else the same.

      ii. The dielectric slab is removed from the capacitor with everything else the same.

      iii. The conducting slab is moved next to the dielectric slab (from $d \le x \le 2d$ to $2d \le x \le 3d$).


metal
slab
dielectric
slab
+
+
+
Q
-
-
-
-Q


E_x
-d
d
2d
3d
4d
5d
6d
x


V
x
-d
d
2d
3d
4d
5d
6d

Points lost: **Total score:** **Grade:**

Breadth:

- Problem 1:
- Problem 2:
- Problem 3:
- Problem 4:
- Problem 5:
- Problem 6:
- Problem 7:
- Problem 8:
- Problem 9:
- Problem 10:
- Problem 11:
- Problem 12:
- Problem 13:
- Problem 14:
- Problem 15:
- Problem 16:
- Problem 17:

Depth

- Problem 1:
- Problem 2:
- Problem 3: