Print your name clearly:____

Signature:____

"I agree to neither give nor receive aid during this exam."

Physics 162 Final Exam

Professor Greenside
Wednesday, April 29, 2015

This exam is closed book and no electronic devices are allowed. Please note the following:

1. There are many shorter breadth questions that come first that are worth 4-6 points each, followed by a smaller number of depth questions that are worth substantially more points. So make sure to pace yourself so that you leave time for the breadth questions.
2. Unlike previous quizzes and exams, **please circle your answers for the multiple choice questions directly on the exam**. Also, just for the multiple choice questions, you do **not** have to justify your answers. However, if you get a multiple-choice answer wrong, you can get some partial credit based on any work you show in the blank space just below each multiple choice question. (Only work written in this blank space will be looked at.)

   All other questions require some justification in order for you to get any credit.
3. Please write your name and the problem number at the top of each extra page, and please sort your pages in order of increasing problem number before handing in your exam.
4. Don't hesitate to ask for help during the exam if you don't understand some question.

Breadth Part 1 (Justification needed, write answers on extra pages)

1. **(6 points)** What are three unsolved scientific questions related to electricity and magnetism that were discussed in class this semester. (You only have to give the questions, no discussion or explanation is necessary.)

2. **(6 points)** Explain briefly, with an appropriate drawing and mentioning relevant equations, why a force is needed to pull a sheet of aluminum foil through the space between the north and south poles of a strong magnet.

3. **(4 points)** One of the key concepts this semester was the idea of an electric field, that a charge fills all of space with some kind of influence and that this influence can exert a force $\mathbf{F}$ on some other remote charged particle with charge q via the rule $\mathbf{F} = q\mathbf{E}$, where $\mathbf{E}$ is the value of the electric field at the location of the charge q. Explain briefly what is the scientific reasoning that justifies treating the electric field as a physical object in its own right, in that it exists independently of the charge that created the field.

4. **(4 points)** True or false and briefly why: an electric field line is a line of constant electric potential.

5. **(4 points)** True or false and briefly why: the electromagnetic waves emitted from Duke's FM radio station WXDU with transmission frequency 88.7 MHz will diffract substantially when passing through a human-size doorway cut out of a wall that is opaque to radio waves.

Breadth Part 2, Multiple Choice (Justification not needed but can help)

6. **(4 points)** Circle each letter that corresponds to a relativistic invariant, which means that the value of the quantity remains the same in all inertial frames of reference. (More than one answer is possible).

   (a) the mass of a point particle
   (b) the charge of a point particle
   (c) the energy of a point particle
   (d) the total force acting on some point particle
   (e) the volume charge density ρ of some object.
   (f) the electric field at some point in space
   (g) the angle between the electric field and magnetic field at some point P.

7. **(4 points)** Assume that Duke broadcasts its FM radio station WXDU using two in-phase antennas that are separated by a distance 2λ equal to twice the wavelength λ of the broadcasted radiowaves. If, with a radio tuned to WXDU, you walk along a circle of large radius $R \gg \lambda$ that is centered on the midpoint between the two antennas, at how many different locations on the circle will you not be able to hear WXDU? (Note: $R \gg \lambda$ means that you can treat line segments from the antennas to any point on the circle as approximately parallel.)

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (f) more than 4

8. **(4 points)** If a 50 W bulb one meter from your face is replaced with a 100 W bulb two meters away, the maximum electric field strength of the light on your face will change by a factor of

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1 (e) $\sqrt{2}$ (f) 2

9. **(4 points)** A straight cylindrical copper wire with conductivity $\sigma \approx 6 \times 10^7\,\Omega^{-1}\mathrm{m}^{-1}$ carries a current of 1 A that is changing at a rate of 10^6 A/s. To the nearest power of ten, the magnitude in amperes of Maxwell's displacement current $\epsilon_0\, d(\Phi_E)/dt$ is:

(a) 10^{-13} (b) 10^{-10} (c) 10^{-7} (d) 10^{-4} (e) 10^{-1}

10. **(4 points)** Unpolarized light of intensity I_0 is incident normal to three successive ideal linear polarizing filters. The transmission axis of the second filter is oriented at 30° with respect to the axis of the first filter, and the axis of the third filter is oriented at 90° to that of the first filter. What is the intensity of light that comes out of the third filter?

(a) 0 (b) $(3/32)I_0$ (c) $(3/16)I_0$ (d) $(1/16)I_0$ (e) some other value.

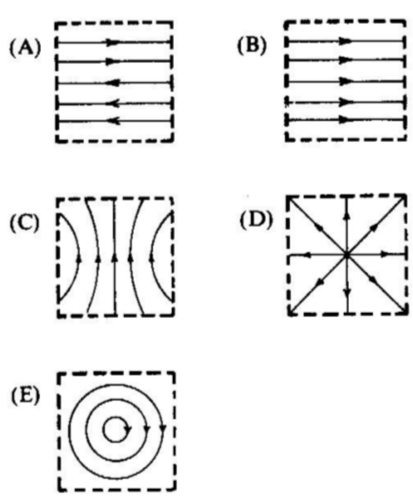
11. **(4 points)** The nearest star to our solar system is about four light years away ($1\,\text{ly} \approx 10^{16}\,\text{m}$). If a planet orbits that star about the same distance as Earth orbits the Sun (one astronomical unit $\approx 1.5 \times 10^{11}\,\text{m}$), to the nearest power of ten, what is the minimum diameter an Earth-based telescope would need to resolve the planet and star, assuming light with wavelength $\lambda = 550\,\text{nm}$?

(a) 1 mm (b) 1 cm (c) 10 cm (d) 1 m (e) 10 m (f) 100 m

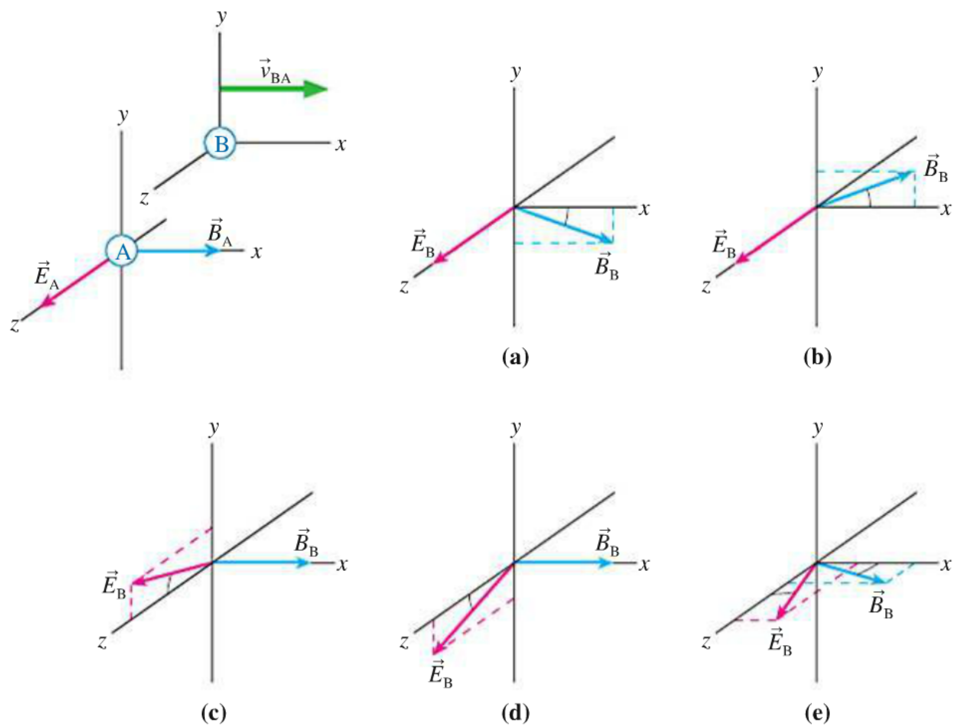
12. **(4 points)** A sphere of radius R has a charge density ρ that varies with radius r as $\rho = Ar^2$ where A is a positive constant. What is the magnitude E of the electric field a distance $R/2$ from the center?

(a) $\dfrac{A}{4\pi\epsilon_0}$ (b) $\dfrac{AR^3}{40\epsilon_0}$ (c) $\dfrac{AR^3}{24\epsilon_0}$ (d) $\dfrac{AR^3}{5\epsilon_0}$ (e) $\dfrac{AR^3}{3\epsilon_0}$

13. **(4 points)** Which one of the following sketches of magnetic field lines is **not** consistent with Maxwell's equation $\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$?



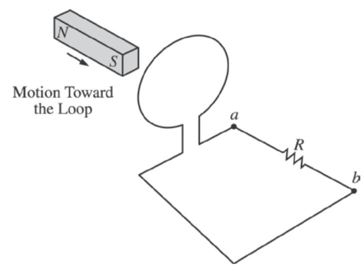
14. The first diagram on the left shows the electric and magnetic fields in a reference frame A. Which diagram shows the fields in frame B, which is moving with a constant velocity $\mathbf{v}_{BA} = v_0\hat{\mathbf{x}}$?



15. **(4 points)** If two inductors with inductances of L_1 and L_2 are connected in parallel, then their equivalent inductance is given by

(a) $L_1 + L_2$ (b) $L_1L_2/(L_1 + L_2)$ (c) $\left(L_1^2 + L_2^2\right)^{1/2}$ (d) some other expression

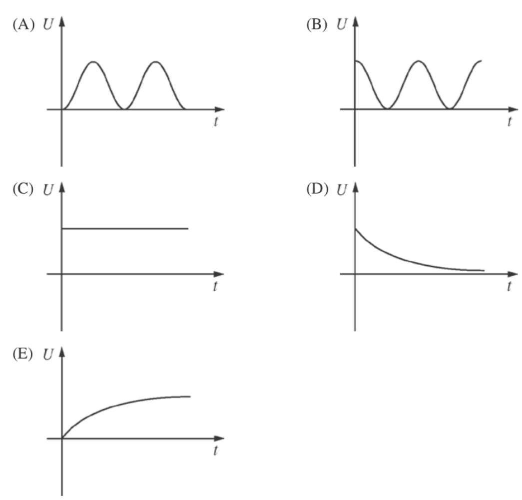
16. Which one of the following statements correctly describes how current flows between points a and b in this figure



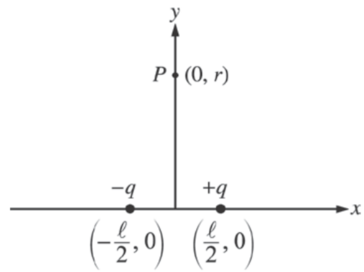
as the bar magnet passes completely through the circular loop in the direction shown?

(a) No current flows between a and b.
(b) Current flows only in the direction from a to b.
(c) Current flows only in the direction from b to a.
(d) Current flows first in the direction from a to b and then reverses direction.
(e) Current flows first in the direction from b to a and then reverses direction.

17. **(4 points)** At time $t = 0$, a fully charged capacitor is connected across the two ends of an inductor. Which of the following curves correctly describes the energy stored in the magnetic field of the inductor as a function of time?



18. **(4 points)** Two point particles with opposite charges $\pm q$ are separated by a distance l as shown in this figure:

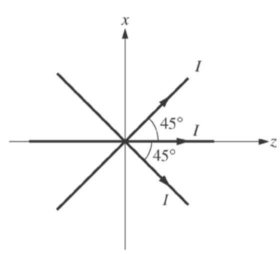


Which one of the following expressions gives the approximate total electric field $\mathbf{E}$ at the point $\mathbf{P} = (0, r)$ on the y-axis that is located a distance r far from the x-axis (so $r \gg l$)?

(a) $\dfrac{2Kq}{r^2}\hat{\mathbf{y}}$ (b) $\dfrac{2Kq}{r^2}\hat{\mathbf{x}}$ (c) $\dfrac{2Kq}{r^2}(-\hat{\mathbf{x}})$

(d) $\dfrac{Kql}{r^3}\hat{\mathbf{y}}$ (e) $\dfrac{Kql}{r^3}\hat{\mathbf{x}}$ (f) $\dfrac{Kql}{r^3}(-\hat{\mathbf{x}})$

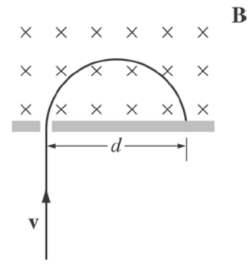
19. **(4 points)** Three long straight insulated wires in the xz-plane carry identical currents I in the directions shown and cross at the origin:



If the coordinates x, y, and z form a right-handed Cartesian coordinate system, then the total magnetic field $\mathbf{B}$ at a point $(x, 0, 0)$ on the x-axis is given by

(a) $\dfrac{3\mu_0 I}{2\pi x}\hat{\mathbf{x}}$ (b) $\dfrac{3\mu_0 I}{2\pi x}\hat{\mathbf{y}}$ (c) $\dfrac{\mu_0 I}{2\pi x}\left(1 + 2\sqrt{2}\right)\hat{\mathbf{y}}$ (d) $\dfrac{\mu_0 I}{2\pi x}\hat{\mathbf{x}}$ (e) $\dfrac{\mu_0 I}{2\pi x}\hat{\mathbf{y}}$

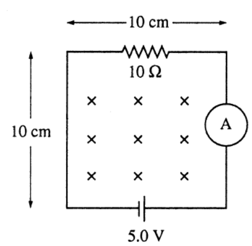
20. **(4 points)** A point particle with mass m, charge q, and velocity $\mathbf{v}$ enters a region of uniform magnetic field $\mathbf{B}$ through a slit in a wall and then strikes the wall a distance d from the slit as shown here:



If the particle's charge-to-mass ratio q/m is doubled while its speed v stays the same, at what new distance from the entrance slit will the particle strike the wall?

(a) $2d$ (b) $\sqrt{2}d$ (c) d (d) $\frac{1}{\sqrt{2}}d$ (e) $\frac{1}{2}d$

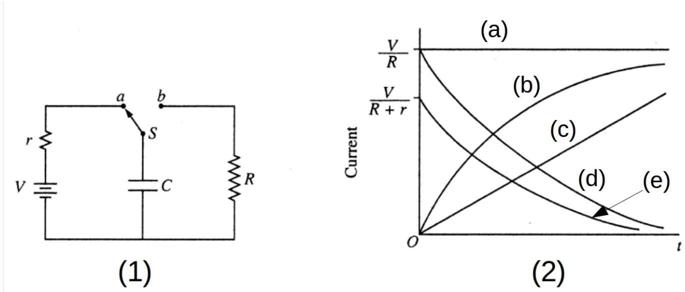
21. **(4 points)** In this circuit



a uniform magnetic field into the page is decreasing in magnitude at the rate of 150 T/s. The reading on the ammeter is

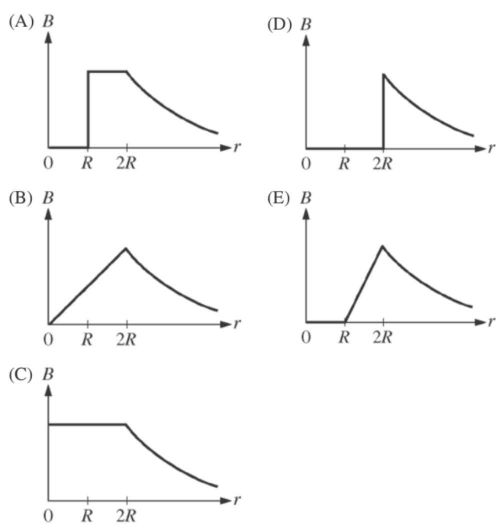
(a) 0.15 A (b) 0.35 A (c) 0.50 A (d) 0.65 A (e) 0.80 A

22. **(4 points)** In panel (1) on the left, a capacitor is charged by connecting switch S to contact a for a long time:

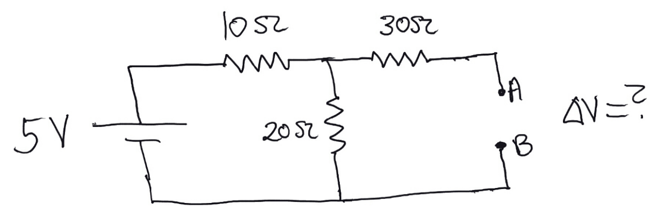


If the switch is then changed to contact b at time $t = 0$, which of the curves in panel (2) on the right correctly describes how the magnitude of the current $I(t)$ through the resistance R changes with time?

23. **(4 points)** A long straight hollow cylindrical conductor with inner radius R and outer radius $2R$ carries a current with a uniform current density. Which of the following graphs best represents how the magnitude of the magnetic field $B(r)$ varies with radial distance r from the axis of the wire?



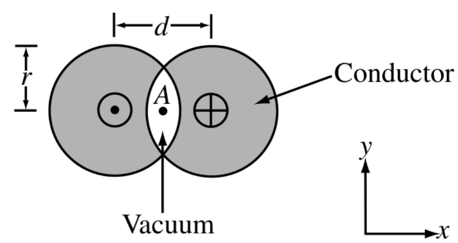
24. **(4 points)** What potential difference $\Delta V = V_A - V_B$ applied by a second battery to points A and B in this circuit



will cause the $20\,\Omega$ resistor to dissipate no power?

(a) 0 V (b) 10 V (c) – 10 V (d) 15V (e) – 15V (f) some other value

25. **(4 points)** Two long straight conductors perpendicular to this page are arranged to form overlapping cylinders each of radius r and with centers that are separated by a distance d. A cross-section of the two conductors looks like this:



A current of current density J (in units of $\mathrm{A/m^2}$) flows into the plane of the page along the shaded part of the cylinder on the right and and an equal current flows out of the page along the shaded region on the left. What is the magnetic field vector $\mathbf{B}$ at the point A in the figure?

(a) $\frac{\mu_0}{2\pi}\pi d J\,\hat{\mathbf{y}}$ (b) $\frac{\mu_0}{2\pi}\frac{d^2 J}{r}\,\hat{\mathbf{y}}$ (c) $\frac{\mu_0}{2\pi}\frac{4d^2 J}{r}\,(-\hat{\mathbf{y}})$ (d) $\frac{\mu_0}{2\pi}\frac{J r^2}{d}\,(-\hat{\mathbf{y}})$ (e) $\mathbf{0}$

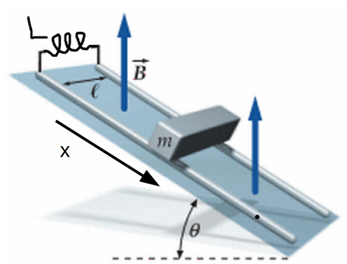
26. **(4 points)** If magnetic charge (isolated north and south magnetic poles) existed in nature, which of Maxwell's four equations

$$
\begin{array}{ll}
\text{I} & \displaystyle\oint_S \mathbf{E} \bullet d\mathbf{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \\
\text{II} & \displaystyle\oint_S \mathbf{B} \bullet d\mathbf{A} = 0 \\
\text{III} & \displaystyle\oint_C \mathbf{E} \bullet d\mathbf{l} = -\frac{d}{dt}\left[\int_S \mathbf{B} \bullet d\mathbf{A}\right] \\
\text{IV} & \displaystyle\oint_C \mathbf{B} \bullet d\mathbf{l} = \mu_0 I_{\text{encl}} + \mu_0\epsilon_0 \frac{d}{dt}\left[\int_S \mathbf{E} \bullet d\mathbf{A}\right]
\end{array}
$$

would have to be changed?

(a) I only (b) II only (c) III only (d) I and IV (e) II and III

27. **(4 points)** A zero-resistance conducting bar of mass m slides down a ramp consisting of two parallel frictionless zero-resistance conducting rails of spacing l in the presence of a vertical uniform magnetic field $\mathbf{B}$. The rails are tilted up an angle θ from the horizon and an ideal inductor with inductance L and zero resistance is connected across the top of the bars as shown here

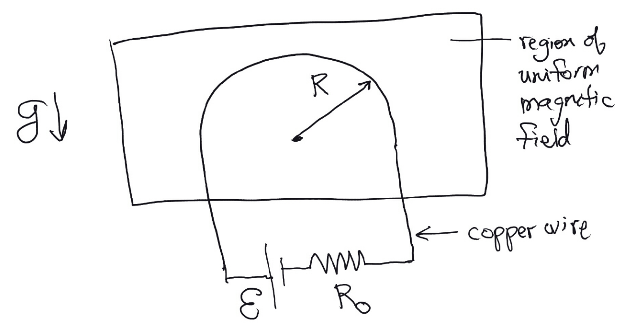


Define an x coordinate along the rails with downhill corresponding to increasing x, in which case $v = dx/dt$ denotes the speed of the bar parallel to the rails. Assuming the bar is released at rest on the rails, what is the form of the differential equation that describes how x evolves, which in turn predicts the qualitative behavior of the bar?

(a) $dv/dt = c_1$, constant acceleration, bar moves ever faster or ever slower
(b) $dv/dt = -c_2 v$, bar initially at rest stays at rest, moving bar comes to halt
(c) $dv/dt = c_3 - c_4 v$, bar speeds up until a terminal speed is attained
(d) $dv/dt = c_5 - c_6 x$, periodic motion about some equilibrium position.

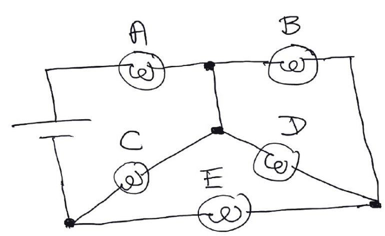
Depth

28. **(10 points)** As shown in this figure



a stiff piece of copper wire is bent into a semi-circle of radius R and connected in series with a battery of emf $\mathcal{E}$ and with a resistor of resistance R_0. The wire, battery, and resistor together have a total mass of M. The wire is then oriented vertically as shown (so down is towards the ground) and placed in a region of space (denoted in cross-section by the rectangle) that is filled with a uniform magnetic field $\mathbf{B}$. Determine the direction and magnitude B of $\mathbf{B}$ so that the copper wire is levitated, i.e., is in mechanical equilibrium with no net force, no net torque. Your expression for B should be in terms of g (acceleration of gravity at the Earth's surface), M, R, $\mathcal{E}$, and R_0.

29. **(10 points)** The following circuit consists of five identical lightbulbs A, B, C, D, and E connected to a battery as shown:

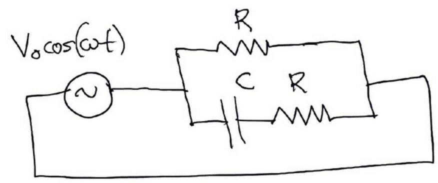


List the the bulbs in *decreasing* order of their brightnesses.

30. (a) **(8 points)** Explain briefly how a two-slit experiment is able to distinguish between the hypothesis that light consists of point-like particles from the hypothesis that light consists of a wave. As part of your answer, sketch how the light intensity function $I(y)$ versus y, as measured on an observation screen, would appear for these two cases.

(b) **(8 points)** Determine to the nearest power of ten what spacing d the two slits should have so that, for monochromatic light with wavelength 500 nm, the five brightest fringes can be observed with spacings of $\Delta y = 2\,\mathrm{mm}$ on a screen that is two meters away from the two slits. Based on that calculation, then determine what is the largest possible width w the two slits can have for the five brightest fringes to even appear on the screen.

31. The circuit shown below consists of two identical resistors with resistance R and a capacitor with capacitance C connected to an AC voltage source $V(t) = V_0 \cos(\omega t)$ where the angular frequency has the specific value $\omega = 1/(RC)$.

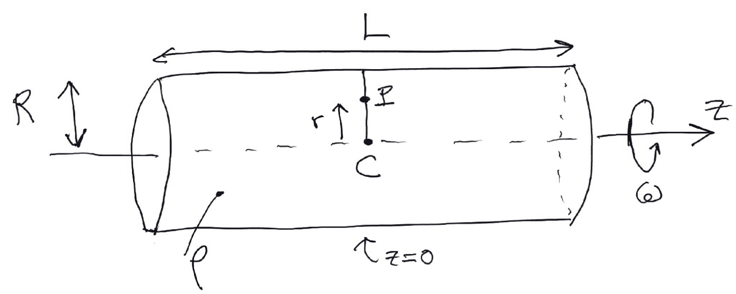


(a) **(5 points)** What is the total complex impedance Z of the circuit? Your answer should be in terms of R only.

(b) **(5 points)** If the total current through the circuit has the form $I_0 \cos(\omega t + \phi)$, what is I_0 and ϕ in terms of V_0 and R?

(c) **(5 points)** What is the average power dissipated by this circuit in terms of V_0 and R? (Evaluate any trigonometric functions explicitly so that your final answer has no trigonometric functions.) Compared to the same circuit without the capacitor, has adding the capacitor increased, decreased, or left unchanged the total power dissipated?

32. Consider a long solid cylinder of radius R and length $L \gg R$ that is made from a non-conducting uniformly charged material with a constant volume charge density $\rho > 0$:



The cylinder is rotating with constant angular speed ω about its axis, which you can assume to be a z coordinate axis, with $z = 0$ being the center of the cylinder. The cylinder is rotating counter-clockwise if you look down the z axis towards the cylinder.

(a) **(4 points)** Far from the rotating cylinder (for $z \gg L$), the magnitudes $E(z)$ and $B(z)$ of the electric and magnetic fields on the z-axis have the simplified form $cz^{-\alpha}$, where c is some positive constant and $\alpha > 0$ is some positive exponent. What are the values of α for $E(z)$ and $B(z)$ and explain your reasoning briefly. Note: no computation is needed for this problem, instead you need to use knowledge from this course to deduce what the exponents must be.

(b) **(8 points)** Draw two qualitative diagrams, one showing the electric field lines inside and outside the rotating cylinder, and a second diagram showing the magnetic field lines inside and outside the rotating cylinder. Also draw some arrows in your two diagrams to indicate the directions of the fields. Note: thinking about parts (c) and (d) below might help to you answer this question.

(c) **(8 points)** As a function of radial distance r, what is the magnitude $E(r)$ of the electric field vector $\mathbf{E}$ along a radial line going from the center C of the cylinder to the cylinder's surface, and what is the direction of $\mathbf{E}$ along this radial line? Also draw a plot of your analytical answer for the magnitude $E(r)$ versus r (for $0 \le r \le R$) to show that you understand its qualitative behavior.

(d) **(18 points)** As a function of radial distance r, what is the magnitude $B(r)$ of the magnetic field vector $\mathbf{B}$ along a radial line going from the center C of the cylinder to its surface, and also what is its direction along this line? Also draw a plot of your analytical answer for the magnitude $B(r)$ versus r (for $0 \le r \le R$) to show that you understand its qualitative behavior.

A hint: instead of decomposing the cylinder into rings or disks, decompose the cylinder into a union of concentric infinitesimally thick rotating shells of radius r, thickness dr, and length L and then argue that each such rotating shell is physically equivalent to a solenoid....

33. **(15 points)** A diffraction grating has 2N parallel slits with equal spacing d, but the slits alternate in width—larger, smaller, larger, smaller—with values of $w(1+\epsilon)$, $w(1-\epsilon)$, $w(1+\epsilon)$, etc where $\epsilon > 0$ is some dimensionless number that is small compared to one, and where w is the average slit width. Deduce and give a formula for the time-averaged light intensity $I(\theta)$ produced on a screen far from the diffraction grating if monochromatic coherent light with wavelength λ impinges normally on the grating, where θ is the angular location of a point on the screen, as measured from a line perpendicular to and bisecting the grating.

Using your formula, describe two ways in which the diffraction pattern of this non-uniform grating differs from the diffraction pattern of a grating with the same spacing d but with uniform widths w, i.e., compare the case $\epsilon > 0$ with the case $\epsilon = 0$: what changes?

Some comments: the alternating widths will cause alternating wave amplitudes so the total wave at some remote point P that is an angle θ away from normal will have the form:

$$\begin{aligned} u_{\text{total}}(P) = \\ & [u_0(1+\epsilon)]\cos(kx-\omega t) \\ & + [u_0(1-\epsilon)]\cos(k(x+\delta)-\omega t) \\ & + [u_0(1+\epsilon)]\cos(k(x+2\delta)-\omega t) \\ & + \ldots \end{aligned}$$

Now express this sum in terms of the real part of a sum of complex exponentials, factor out common factors, and—as one extra step—collect all terms that have a coefficient of ϵ. You should then be able to complete the calculation.

Breadth points lost: Total Score Exam grade:

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