Physics 55 Midterm Exam: Answers

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This 75-minute exam is closed book and no electronic aids are allowed except for a scientific calculator. If you have any questions during the exam, please do not hesitate to ask me. The following data and equations may be helpful:

$$p^2 = a^3, \qquad p^2 = \frac{4\pi^2}{G(M_1+M_2)}a^3, \qquad M = \frac{rv^2}{G}, \qquad F = G\frac{M_1M_2}{d^2}, \qquad g = \frac{GM}{d^2}, \qquad v = \sqrt{\frac{2GM}{r}}. \quad (1)$$

$$n = N/V = 1/d^3, \qquad a = (1/m)F, \qquad \text{sum } mv \text{ before} = \text{sum } mv \text{ after}, \qquad C = 2\pi r. \quad (2)$$

$$G \approx 7\times10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}, \qquad c \approx 3\times10^8\,\frac{\mathrm{m}}{\mathrm{s}}, \qquad M_{\mathrm{Earth}} \approx 6\times10^{24}\,\mathrm{kg}, \qquad \mathrm{M}_{\mathrm{Moon}} \approx 7\times10^{22}\,\mathrm{kg}. \quad (3)$$

$$R_{\mathrm{Earth}} \approx 6,000\,\mathrm{km}, \qquad \mathrm{R}_{\mathrm{Moon}} \approx 2,000\,\mathrm{km}, \qquad \mathrm{d}_{\mathrm{E-M}} = \text{Earth-Moon distance} \approx 4\times10^5\,\mathrm{km}. \quad (4)$$

True or False Questions (1 point each)

Please circle "T"or "F" to indicate respectively whether each statement is true or false.

1. T / F The Sun's diameter is about 100 times bigger than the Earth's diameter.

   Answer: T. This is one of about ten important facts about sizes and time scales from Chapter 1 that the class should remember after the course is over, e.g., the relative size of some planets (Jupiter is about ten times bigger than Earth), that the orbit of the Moon can easily fit inside the Sun, the size of the solar system out to Pluto (40 AU, 6 light-hours), distances between stars, distances between galaxies, sizes of galaxies, and so on.

2. T / F Mass and weight have different physical units.

   Answer: T. The mass of some object is a number that measures the resistance of the object to acceleration when a force is applied to the object (a big mass means small acceleration) and has units of kilograms (kg). The weight of some object is the total force acting on the object, some of which is due to gravity. As a force, weight is a vector or arrow and so has two attributes, the direction in which the force is acting on the object and the strength or magnitude of the force. The strength is measured in units of newtons (N) and has fundamental units of "mass times acceleration" or $\mathrm{kg\,m/s^2}$ (which follows from Newton's second law in the form $\mathbf{F} = m\mathbf{a}$).

   More generally, the units of some quantity in terms of the fundamental units of length (meters m), mass (kilograms kg), and time (seconds s) can be obtained by using basic physics formulas. For example, the equation for momentum $p = mv$ (I don't know the origin of how the letter p was chosen for momentum, perhaps related to the word puissance) tells us that the units of momentum is the product of the units for mass and speed respectively: $\mathrm{kg} \times (\mathrm{m/s})$ or $\mathrm{kg\,m/s}$. Similarly, the complicated units for the gravitational constant G can be deduced by rewriting Newton's universal law for gravity $F = GM_1M_2/d^2$ in the form $G = Fd^2/(M_1M_2)$ and substituting the units for force: $(\mathrm{kg\,m/s^2})\,\mathrm{m^2/kg^2} = \mathrm{m^3/(kg\,s^2)}$.

3. T / F If the Earth did not have a moon, there would still be tides.

   Answer: T. The tides on Earth come from the tidal forces of the Moon and from the tidal forces of the Sun, with the latter being about one-third as strong as the Moon. So if the Earth did not have a Moon, there would still be tides and high tide would still occur about once every 12 hours since it is the fast rotation of the Earth through the bulges caused by the Sun that would cause someone on Earth to see high and low tides. The magnitude of the tides (how high the water rises during high tide) would be less and there would no longer be neap or spring tides.

4. T / F The Sun is constantly falling toward the Earth.

   Answer: T. As we learned when discussing Newton's laws of motion combined with Newton's universal law of gravity, the Sun and Earth, as a binary system gravitational system, both pull on each other with equal and opposite forces and so the Sun is technically falling toward the Earth just as the Earth is constantly falling toward the Sun. More specifically, the Sun and Earth are both following elliptical orbits about their mutual center of mass (which lies deep inside the Sun since the mass of the Earth is about a millionth the mass of the Sun) which lies at one focus of their orbits. But without the gravitational pull of Earth on the Sun, it would not follow this orbit and move in a straight line at constant speed.

5. T / F Right ascension is measured in solar time.

   Answer: F. Right ascension is indeed measured in units of time, rather than in angular degrees like declination, but it is measured in sidereal time since right ascension concerns the location of objects on the celestial sphere and the sphere rotates once exactly in a sidereal day of 34 sidereal hours (which corresponds to a solar time of $23^{\mathrm{h}}\,56^{\mathrm{m}}\,4.01^{\mathrm{s}}$).

6. T / F If you experience weightlessness, then your weight is zero.

   Answer: F. Weightlessness is the term used for free fall, when an object is moving about in response to gravity without other forces present. (Examples of other forces might be the push from a rocket engine, an electrical attraction or repulsion, or the floor pushing you up so that you don't fall.) Since the acceleration of an object under gravity does not depend on the mass of the object (this is Galileo's famous observation that a feather and lead weight would hit the Earth at the same time when dropped from a tall tower if air could be ignored, and follows from the equation $F = ma = GmM/d^2$ or $a = GM/d^2$), all objects fall together in the same way or orbit the Earth in the same way. Thus people and objects in the Space Shuttle (when the Shuttle is in orbit around the Earth) are weightless and float around since there is no relative acceleration of one object with respect to another, they are all falling in the same way around the Earth.

   The weights of people and objects in the Space Shuttle are not zero, they are still being pulled toward Earth by a substantial force equal to GmM/d^2 where d is the distance from the center of the Earth to the Space Shuttle.

7. T / F If a car drives along a circular racetrack at a constant speed of 80 km/h, then the car is accelerating.

   Answer: T. Acceleration is nonuniform motion, which means a change in speed (speeding up or slowing down) or a change in direction, or both. Constant speed around a circular path is an important astronomical example of acceleration since the orbits of the planets are approximately circles (with the exception of Pluto, whose orbit has a substantial eccentricity).

8. T / F All objects on the celestial sphere with the same right ascension will rise above your local horizon at the same time.

   Answer: F. It is true that all objects that rise above your horizon at the same time have the same right ascension since your horizon is parallel to great arcs of constant right ascension (see Fig. S1.15b on page 100 of the text). But if you are anywhere other than at the equator, you can not see all the objects with the same right ascension since some of them will be hidden below your local horizon. As an example, someone at the North Pole can only see objects with declinations between +0 and +90 degrees and so can't see objects below the celestial equator, which means about about half the objects that have the same right ascension as those above the celestial equator.

9. T / F If the total force on an object is zero, then the speed of the object must be zero.

   Answer: F. If the total force on an object is zero, the object has zero acceleration which means that its velocity is uniform, always pointing in the same direction and not changing in value. But this value can be nonzero, for example a puck sliding across an air table.

10. T / F It is possible for someone at the North pole and someone at the South pole to see the same star (although not necessarily on the same day.)

    Answer: T. Stars are at such immense distances from the Earth that the light from a star basically strikes the Earth in parallel lines. So the same star can be visible from the North and South pole, although not necessarily on the same day since it may be daytime at the North pole (no stars visible) while it could be nighttime at the South pole. A simple example would be the Sun itself on the fall or spring equinox in which case the Sun would be exactly on the horizon of the local sky at the North and South pole.

11. T / F A comet approaching the Sun along the path of a hyperbola will eventually leave the solar system and not return.

    Answer: T. We learned in lecture and from our text that Newton, using his equations of motion and his law of gravity, discovered that objects could orbit the Sun in shapes other than ellipses, namely parabolas and hyperbolas. These latter two shapes differ from ellipses in describing *unbound* orbits, i.e., orbits that will carry the object arbitrarily far from the Sun without a return. All three orbits are conics, curves obtained by slicing a cone at different angles with a plane.

12. T / F The orbital velocity law allows you to estimate the total amount of mass that lies within the circular orbit of some object whose mass is small compared to the amount of mass inside the circular orbit.

    Answer: T. This is, in fact, how the orbital velocity law would be described. It is important to appreciate the qualification that the mass of the object m orbiting in some cloud of other objects has to be small compared to the total mass enclosed by the orbit. Otherwise, in Newton's version of Kepler's law that $p^2 = 4\pi^2 a^3/(G(m+M))$, all we could deduce is the sum of the masses $m+M$, not the mass M of enclosed by the orbit.

13. T / F If moon A of a given planet is less massive and hotter than moon B of the same planet, then moon A is *less* likely to have an atmosphere than moon B.

    Answer: F. I discussed in lecture that the escape velocity formula $v = \sqrt{2GM/r} = (2GM/r)^{1/2}$ gave a valuable insight into whether a planet or moon might have an atmosphere: the warmer the surface of the planet, the larger the average speed of molecules in the atmosphere and it may then be possible for a sizable fraction of the molecules to exceed the escape speed v of the planet in which case the atmosphere leaks off into space. (The fact that atoms or molecules are very tiny masses is irrelevant, the same escape speed formula holds.) From the formula, we see that the escape speed depends on the mass M of the planet *and* on the radius r of the planet. (Most atmospheres are thin compared to the size of a planet so all molecules in the atmosphere are basically one radius away from the center of the planet.)

It is then true that, for a fixed radius r, making a moon less massive (decreasing M) will decrease the escape speed v and making a moon hotter will make it easier for the atmosphere to escape from that particular moon. But if we are to compare one moon with another, we also have to consider the radius r of each moon, and then we realize that making a moon less massive and hotter could be compensated for by making the radius of the moon much smaller, in which case the escape speed could become big. (This is what happens for neutron stars and black holes, the mass stays constant but the size becomes so tiny that the escape speed approaches the speed of light.) So this true/false statement would be true for two moons of the *same* radius, but generally can not be expected to hold for moons of different sizes.

14. T / F If the Moon were changed into a black hole of equal mass (such a black hole would be about 0.1 mm in size) without altering its orbit, then the height and frequency of the tides on Earth would not change.

    Answer: T. The Earth's tides arise from the Moon's tidal forces, which in turn depend only on the distance of the the center of mass of the Moon to the various parts of the Earth. (You saw an example of this in Assignment 5, when you calculated the forces of the Moon on 1 kg rocks on the close and far sides of the Earth compared to the Moon.) So changing the size of the Moon, in particular making it quite tiny, doesn't change anything, the Moon would orbit as before and the tides would remain unchanged.

15. T / F For any two comets orbiting the Sun in elliptical orbits, the comet whose aphelion is further from the Sun will have the longer orbital period.

    Answer: F. From Kepler's law $p^2 = a^3$, the orbital period p depends on the semimajor axis a, which in turn depends on the planet's distance at aphelion *and* on the distance at perihelion since the semimajor axis is the average of these distances. Thus knowledge of just the distance at aphelion is not enough to determine a and a larger value does not automatically imply a longer period.

    As a specific example, consider one comet in a large circular orbit of radius R centered on the Sun, and consider a second comet in a highly-flattened elliptical orbit of large eccentricity with the Sun at one focus such that the comet comes very close to the Sun at perihelion (say just outside the surface of the Sun) and goes just a bit outside the circular orbit at aphelion. Then the second orbit would have the larger distance at aphelion but would have a semimajor axis close to $R/2$ in value in which case the comet with the circular orbit, which has the smaller distance at aphelion, would have the longer orbital period p.

16. T / F If the Earth had many moons in addition to the Moon, all orbiting in the ecliptic plane with approximately circular orbits and in the same direction as the Moon, then the moon whose orbit has the smallest semimajor axis would have the smallest difference between its synodic month and its sidereal month.

    Answer: T. See Figure S1.3 on page 89 of the text: the faster a moon orbits about the Earth, the less time there is for the Earth to move some distance along its orbit about the Sun, the smaller the extra angle through which the Moon has to move to line up with the Sun again, and so the closer the synodic and sidereal months are in value. Since we know from Newton's version of Kepler's third law that small periods p are associated with smaller semimajor axes a, the moon closest to Earth with the smallest a will have the shortest period and so the closest values between its synodic and sidereal months.

    For example, in the extreme case of a moon orbiting 30 times faster than today's Moon, about once a day, the Moon would be rotating together with the Earth so a synodic month would be the same as a solar day and a sidereal month would be the same as a sidereal day, and the difference between a synodic and sidereal month would be about 4 minutes, much shorter than the present difference of about two days. You should be able to show that the Moon would have to then orbit at a distance of about 42,000 km (26,000 miles) from the center of the Earth to achieve this synchronous period, i.e, about nine times closer to Earth than its present orbit.

Multiple Choice Questions (4 points each)

Circle the letter that best answers each of the following questions.

1. Between the fall equinox and winter solstice,

   (a) the Sun rises south of due east and sets south of due west.
   (b) the Sun rises south of due east and sets north of due west.
   (c) the Sun rises north of due east and sets north of due west.
   (d) the Sun rises north of due east and sets south of due west.
   (e) the Sun rises due east and sets due west.

   Answer: (a). Where an object rises and sets depends purely on its declination above or below the celestial equator. Since the celestial equator always passes exactly through the east and west points of your local horizon except at the poles (study the figures on pages 99-101 of the text), and because on any given day, an object on the celestial sphere rotates through a circle parallel to the celestial equator, any object with a positive declination rises north of east and sets north of west, and any object with a negative declination rises south of east and sets south of west. Between the fall equinox and spring equinox, not just between the fall equinox and winter solstice, the Sun has a negative declination and so (a) is the answer.

2. If the Moon had a liquid ocean, then someone at a fixed location on the Moon's equator

   (a) would not see any tides.
   (b) would see high tides about once a day.
   (c) would see high tides about once a week.
   (d) would see high tides about once every two weeks.
   (e) would see high tides about once a month.

   Answer: (d). Just as the tides on Earth are affected by the tidal forces of the Moon and Sun (with the latter being about one third as strong as the effect of the Moon), an ocean on the Moon would feel tidal forces from the Earth and Sun. Other planets such as Venus, Mars, and Jupiter have either too small a mass or are too far away to produce significant tides.

   Let's consider the effects of the Earth and Sun in turn. Since the Earth is about 80 times more massive than the Moon, it will cause considerably larger bulges of the Moon's ocean compared to Earth's tides, with one bulge facing the Earth and an equal-size bulge on the far side of the Moon (see Figure 5.15 on page 143 of the text). But here is a big difference: the Moon turns once on its axis exactly in the same time that it orbits once around the Earth so that the same side of the Moon always faces the Earth. (Read the section "The View from the Moon" on pages 43-44 of the text, the Earth sits motionless in the local sky of someone on the Moon.) This means that the Moon would not rotate through the bulges of its oceans as is the case for Earth, and so the tidal bulges on the Moon just due to the Earth do not change with time for someone at a fixed location on the Moon's equator, i.e., there are no high tides or low tides because of the Earth.

   But the Moon has a second smaller tidal bulge that is aligned with the direction of the Moon to the Sun. Since the Moon is indeed rotating about its axis once a sidereal month of $27\frac{1}{3}$days (see Fig. 2.22 on page 43 of the text), and since this rotation time is short compared to the time for the Moon to orbit the Sun (which is also a year like the Earth), someone on the Moon's equator will see weak tides rise and ebb about once every two weeks because of the Sun. So the answer is (d).

3. The time between the rising and setting of a star
   (a) is always 12 sidereal hours.
   (b) depends on the star's right ascension.
   (c) depends on the observer's latitude.
   (d) depends on the observer's longitude.

   Answer: (c). We saw above in multiple choice problem 1 that where the Sun or any other object rises and sets depends on its declination. Similarly, how long the object spends above the horizon—the amount of time between rising and setting—depends on the object's declination and on the observer's latitude (see Figure S1.15 on page 100 of the text). An object rises and falls in 12 sidereal hours only if its dec is $0°$. A star's right ascension determines at what sidereal time the star will rise above your local horizon but does not determine how much time it spends above the horizon. Similarly, an observer's longitude also determines when a star will rise and set but not the difference in time between these events. So (c).

4. Precession refers to
   (a) the order of planets along the ecliptic.
   (b) the change in the direction of the Earth's axis over a 26,000 year period.
   (c) the change in the tilt of the Earth's axis over a 26,000 year period.
   (d) the change in position of an object against a background of stars because of changes in the location of the Earth during its yearly orbit.
   (e) the reversing of direction of a planet during retrograde motion.

   Answer: (b). Precession is a slow circling of the direction, but not the tilt, of the Earth's rotation axis. It is caused by the pull of the Moon and Sun on the slightly non-spherical bulge of the Earth at its equator that arises from the Earth's rotation. The direction, but not tilt, changes for reasons that go beyond the level of Physics 55 but are associated with the conservation of angular momentum, that the direction and strength of some kind of rotation can be changed only by having a nonzero total torque on the object.

5. Parallax refers to
   (a) the order of planets along the ecliptic.
   (b) the change in the direction of the Earth's axis over 26,000 years.
   (c) the change in the tilt of the Earth's axis over 26,000 years.
   (d) the change in position of an object against a background of stars because of changes in the location of the Earth during its yearly orbit.
   (e) the reversing of direction of a planet during retrograde motion.

   Answer: (d). This is a concept that I did not mention in lecture but that was prominent in Chapter 2 (e.g., in the "Summary of Key Concepts" on page 53 of the text, also Section 2.6) since it is the main reason why the Greeks and others rejected the idea that the Earth orbits about the Sun. Basically, if the stars were close enough to Earth, they would undergo motion relative to stellar objects much further away as the Earth moved to different parts of space in an orbit about the stars (this is similar to why retrograde motion occurs). Since the Greeks could not detect such motion, there was no observable parallax, and because they could not imagine the stars being effectively infinitely far away (a light year would have been beyond their comprehension), they concluded that the Earth was the center of the universe and that the stars, Sun, and planets orbited the Earth at some large but finite distance.

Parallax is one of a few key concepts that I will expect you to learn on you own by reading the text. The Summary of Key Concepts at the end of each chapter is the first place to look when preparing for quizzes or exams.

6. An error of one degree in latitude (corresponding to the angular distance of the star Polaris from the north celestial pole) corresponds to an error of one's location on the Earth's surface of about

   (a) 1 km.

   (b) 10 km.

   (c) 100 km.

   (d) 1000 km.

   Answer: (c). A great circle of the Earth such as a circle of constant longitude spans a circumference of $C = 2\pi R_{\rm Earth}$ and also spans 360°. So the length per degree for longitude would be

   $$\frac{2\pi R_{\rm Earth}}{360^\circ} \approx \frac{2\pi(6\times 10^3\,{\rm km})}{360^\circ} \approx 105\,\frac{\rm km}{^\circ}. \tag{5}$$

   So an error of one degree in latitude or longitude corresponds to not knowing where you are to better than about 100 km. This is a pretty big distance if you are lost in a boat or in a desert.

7. If two black holes A and B of equal mass collide and merge to form a new black hole, then the radius of the new black hole will be about

   (a) half the radius of black hole A.

   (b) twice the radius of black hole A.

   (c) 0.71 times the radius of black hole A.

   (d) 1.4 times the radius of black hole A.

   Answer: (b). We learned in class that a formula for the radius of a black hole, its so-called Schwarzschild radius, could be obtained by setting the escape speed v for some mass M to the maximum possible speed which is the speed of light c. So

   $$c = \sqrt{\frac{2GM}{R}} \quad \Rightarrow \quad R = \frac{2GM}{c^2}. \tag{6}$$

   Thus the radius R of a black hole is proportional to its mass M. If two identical black holes collide, the total mass of the new black hole will be $2M$ and the radius will be twice as big as either of the two black holes that collided.

   This is a rather different result than if two identical planets of mass M collided to form a new spherical planet of mass $2M$, if we also assume that the material of the planets is incompressible like rock and iron (the inner quarter of the Earth is a giant ball of iron bigger than the entire Moon, see Figure 10.2 on page 258 of the text). Then the total volume of the new planet would be twice the original volume, which would mean that the radius would be $2^{1/3} \approx 1.3$ times bigger, which is substantially less than a doubling in radius as is the case for black holes. So black holes are rather strange beasts, they take up more space when they collide than they would if they were liquid droplets that merged their volumes after colliding.

8. The rotational axis of Mars is tilted 24°, about the same as Earth's, but unlike Earth the distance between Mars and the Sun varies considerably during a Martian year. When the martian southern hemisphere is tilted toward the Sun, Mars is about 20% closer to the Sun than when the martian northern hemisphere is tilted toward the Sun. Summers in the martian northern hemisphere are therefore

(a) colder than summers in the martian southern hemisphere.

(b) warmer than summers in the martian southern hemisphere.

(c) about the same as summers in the martian southern hemisphere.

Answer: (a). I took this question directly from a quiz question in the Astronomy Place tutorial on seasons, and will take some future quiz and exam questions from other tutorials. The discussion on pages 38-39 of the text answers this question: unlike Earth, Mars is much closer to the Sun in wintertime so whichever hemisphere is facing the Sun at that time, the southern according to this problem, will be substantially warmer than the opposite hemisphere when Mars reaches aphelion.

9. Black holes are especially dangerous for space travelers because

   (a) their escape speed is greater than the speed of light.

   (b) they suck everything into them.

   (c) they are impossible to detect.

   (d) they can cause enormous tidal forces.

   Answer: (d). Black holes are dangerous compared to planets, asteroids, or stars because they are so tiny in size that an object can get rather close to its center of mass in which case the distance d in the Newton force formula $F = GMm/d^2$ can be quite small, leading to tremendous tidal forces that can rip an object apart. (When an object is close to a black hole or neutron star, there is a huge variation of the strength of gravity from one end of the object to the other, and this variation pulls the object apart.)

   The answer is not (a) since a huge escape speed by itself just means that it is difficult to escape a black hole once you are in orbit near one, the large escape speed does not itself cause harm to a person or spaceship. Black holes do not suck things into them, they are really no different than any regular massive object like the Earth, Moon, or Sun in terms of orbital motion. It is definitely possible to detect black holes: their enormous gravity can make stars have an extremely high speed orbit with a small semimajor axis (this is how the huge black holes at the centers of galaxies are detected), or there is an accretion disk in which material from a nearby binary star spirals in to the black hole, heating up by friction along the way and so emitting lots of light.

10. In the future, when the width of our expanding universe has grown to 500 billion light years (5×10^{11} ly), what will be the ratio of the average separation of galaxies to the average size of a galaxy? Assume that the total number of galaxies will be the same number 10^{11} as observed today, that the galaxies are approximately spread out uniformly in space, that the universe is approximately cubic in shape, and that the typical size of a galaxy will remain about 100,000 ly.

    (a) about 10.

    (b) about 100.

    (c) about 1,000.

    (d) about 10,000.

    (e) about 100,000.

    Answer: (c). This problem was a variation of an earlier homework problem, and depends on the insight that, if a cloud of many objects is spread roughly uniformly throughout space, the density of objects n (number per unit volume) determines their average separation d and vice versa. From the information in the problem, the number density of galaxies in the far future will be

$$n = \frac{N}{V} = \frac{10^{11}\,\text{galaxies}}{(500 \times 10^9\,\text{ly})^3}. \quad (7)$$

The average distance between uniformly distributed galaxies is then about

$$d = \left(\frac{1}{n}\right)^{1/3} = \left(\frac{V}{N}\right)^{1/3} = \frac{V^{1/3}}{N^{1/3}} = \frac{5 \times 10^{11}\,\mathrm{ly}}{(10^{11})^{1/3}} \approx 1.1 \times 10^{8}\,\mathrm{ly}. \tag{8}$$

Dividing this by the typical size of a galaxy 10^5 ly gives a ratio of about 1,000, much bigger than the present day ratio of about 20.

11. Hockey player A (mass 130 kg) is skating from the west with speed 15 km/h toward hockey player B (mass 160 kg) who is stationary on the ice with control of the puck. Hockey player C (mass 110 kg) is skating from the east with speed 20 km/h toward B to get the puck. If all three hockey players collide at the same time and grab onto each other, in what direction will the three players be moving and with what speed? Assume that the players slide without friction after they collide (e.g., they knock each other down and are sliding on their backs in a pile).

    (a) 0.63 km/h to the west.
    (b) 0.63 km/h to the east.
    (c) 1.0 km/h to the west.
    (d) 1.0 km/h to the east.
    (e) the hockey players will be motionless after the collision.

    Answer: **(a)**. This is a conservation of momentum problem similar to the one discussed in lecture (bullet hitting a soft wood block) and to the Calvin and Hobbes problem of a recent quiz. You need to equate the sum of all momenta before a collision to the sum of all momenta after a collision. You also have to make an arbitrary choice about what a positive velocity means, say we choose positive to mean going to the east (left to right) and negative meaning the opposite. We then recognize that there are three masses that collide. The total initial momentum is:

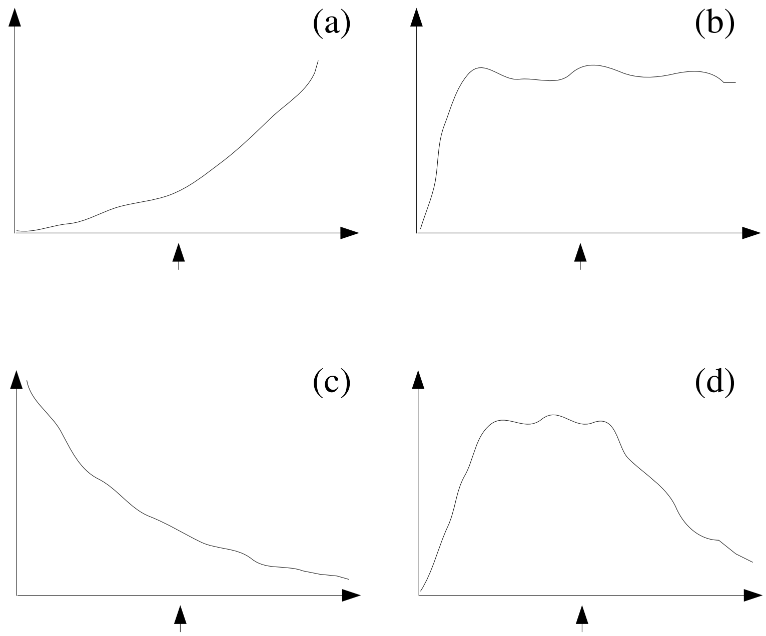
$$m_1 v_1 + m_2 v_2 + m_3 v_3 = (130\,\mathrm{kg} \times 13\,\mathrm{km/h}) + (160\,\mathrm{kg} \times 0\,\mathrm{km/h}) + (110\,\mathrm{kg} \times (-20\,\mathrm{km/h})). \tag{9}$$

    This has to equal the total momentum after the collision, which is some unknown final speed v times the mass $M = m_1 + m_2 + m_3$ of all the three hockey players in a big rigid pile. Equating

$$(130 + 160 + 110)v = (130\,\mathrm{kg} \times 13\,\mathrm{km/h}) + (160\,\mathrm{kg} \times 0\,\mathrm{km/h}) + (110\,\mathrm{kg} \times -20\,\mathrm{km/h}), \tag{10}$$

    and solving for v, one finds that $v \approx -0.63\,\mathrm{km/h}$. Since the speed is negative, all three players are sliding to the left, i.e., to the west. You would get an answer of 1.0 km/h if you left out the mass of the 160 kg player B.

    Note that there is no need to spend time converting velocity units from km/h to, say, m/s. If you look over the above math, you should be able to see that the speed v has to have the speed units of the right side (you are adding quantities of kg times km/h, then dividing by kg), so the answer is directly in km/h, which is useful since you can then compare directly the final speed v with the initial speeds of the players, e.g., 0.63 km/h is rather small compared to the initial speeds of players A and C.



12. The following questions relate to the above figure, which shows rotation curves for four hypothetical galaxies comparable in size to the Milky Way. The vertical arrow roughly midway and beneath each horizontal axis indicates the point beyond which to the right the stars of the galaxy are no longer visible.

    (a) **(2 points)** Label the horizontal and vertical axes of panel (a) with the names of the quantities associated with each axis.

    Answer: A rotation curve for some rotating object gives information about how the speeds of different parts of the object vary with distance from the axis of rotation of the object (the axis in turn generally passes through the center of mass of the object). The plot is especially interesting for objects that are composed of many parts that do not rotate rigidly together, e.g., a galaxy consisting of many stars. For a galaxy, the horizontal axis would be the distance from the center of a galaxy to some point P of interest out in space. The vertical axis is the speed of matter at the location P.

    Note: There will be only a few plots that the class has to learn during this semester, three at most. When we come to these plots (of which the rotation curve is the first), you should study them carefully and memorize a few simple details such as what variables are being plotted, the physical units of the variables, the ranges of values for the variables, and what kind of information can be obtained from the plot.

    (b) **(2 points)** Label the horizontal and vertical axes of panel (b) with the physical units associated with each axis.

    Answer: The horizontal axis could be any unit related to length, such as meters, kilometers, AU, or light years. However, given that we are talking about galaxies, a good choice for the length unit would be light years (ly) since a galaxy has a radial extent of about 50,000 ly.

    The vertical axis could be any unit corresponding to speed, for example m/s, km/h, or some fraction of the speed of light.

    (c) **(1 points)** Indicate the approximate numerical range of the horizontal axis.

    Answer: Since a large spiral galaxy has a diameter of about 100,000 ly as we learned from Chapter 1, the distance out to the little arrow midway across the horizontal axis would be the radius of a galaxy, 50,000 ly, so a numerical range for the entire axis would be from 0 ly to 100,000 ly.

I did not ask for the numerical range of the vertical axis since there is no reason why that range would have special significance and stick in your mind. But galactic rotation speeds can be as large as 300 km/s, which is about $0.001c$, pretty fast!

(d) **(4 points)** Circle the letter that corresponds to a galaxy surrounded by dark matter.
(a) (b) (c) (d).

Answer: (b). If the mass of a galaxy consisted only of its stars, as was believed before observations were carried out, then the total mass enclosed in some large circle of radius r, centered on the center of the galaxy, would become constant once the radius r exceeded the radius of the galaxy since no new mass would be picked up beyond the observable edge of the galaxy. By the orbital velocity law on page 614 of the text, $M_r = rv^2/G$, if M_r becomes constant, then the speed v of matter orbiting about the galaxy at distance r from the galaxy's center must decrease as r increases so the curve would like curve (d). But instead, experiments show the unexpected curve (b), the speed does not decrease to the right of the arrow which is the visible edge of the galaxy, but instead stays roughly constant far beyond the edge of the galaxy. This can happen only if the amount of mass enclosed in a big circular orbit of radius r continues to increase beyond the range of visible stars, which in turn implies the existence of some kind of dark matter.

(e) **(4 points)** Circle the letter that corresponds to a galaxy if dark matter did not exist.
(a) (b) (c) (d).

Answer: (d). This was explained in the discussion for the immediately previous problem.

(f) **(4 points)** Circle the letter that corresponds to a galaxy such that the center of the galaxy is an enormous black hole whose mass greatly exceeds the combined mass of all the other stars in the galaxy.
(a) (b) (c) (d).

Answer: (c). If the galaxy has a black hole of enormous mass, much greater than the mass of all the remaining stars in the galaxy, the rotation curve will be similar to that obtained for our solar system for which the Sun is by far the largest amount of mass: the enclosed mass M_r is approximately constant once r is bigger than the radius of the Sun and the orbital velocity law $M_r = rv^2/G$ then implies that v must decrease with increasing r (in fact, v decreases as $1/\sqrt{r}$ with increasing r).

Open Questions

For the following **five** questions (with one exception), you must justify your answers briefly to get credit. If the grader and I can not *easily* read and understand your answers, you will lose credit.

1. **(5 points)** Name one astronomical event that appeared in the news (newspapers, magazines, TV, radio, Internet, Physics 55 announcements) since the semester began and explain briefly why the event warranted mention in the news.

   There was no unique answer here. Way too many students, over half, were not able to mention a significant astronomical event that appeared in the news since the semester began, many of which I listed in the weekly announcements to the course. Some students mentioned events like the fall equinox on September 21 or the close proximity of Venus and Jupiter in the sky around middle September, but those were not events representing some significant discovery or political activity related to astronomy and certainly did not appear as major news articles over the semester.

At least for this semester but even after this semester, please try to stay aware of ongoing astronomical discoveries and events, there are typically several each week. Besides newspapers and the Internet, a great place to learn about astronomical events is through the "Astronomy Picture of the Day", to which there is a link on the Physics 55 home page. Independent of the fact that these discoveries represent an expenditure of your tax dollars, these discoveries are often genuinely exciting: we humans are continuing to make progress in exploring and sometimes understanding this huge mysterious universe that we live in. Further, you are increasingly able to appreciate at a technical level how some discoveries were made or the significance of some discoveries.

For example, there were several news articles over the last month about the discovery of a 10th planet out in the Oort cloud (unofficially named "Xena") and that even a moon had been discovered orbiting this distant object (the moon has been unofficially named Gabrielle, after the name of the sidekick of television star Xena the Warrior Princess). You are now able to appreciate what is the Oort cloud and how the large distance of Xena from the Sun, its semimajor axis a, determines how long it takes for Xena to orbit the Sun. Further, you can now appreciate that by observing how long it takes Gabrielle to orbit Xena and how far Gabrielle is from Xena on average, we can deduce the mass of Xena via Newton's version of Kepler's third law. From the mass and radius of Xena, we can deduce its mass density $\rho = M/[(4/3)\pi r^3]$ which then tells us something about what Xena is made of, e.g., a density larger than the density of water (1,000 $\mathrm{kg/m^3}$) would imply a rocky planet, close to the density water would imply an icy comet-like object, and less than water would imply a gaseous planet like Uranus or Neptune.

2. **(5 points)** If the celestial coordinates of a certain star are RA $15^{\mathrm{h}}\,42^{\mathrm{m}}$ and dec $-26^\circ\,52'$, determine the coordinates of the point on the celestial sphere that is as far away as possible from this star.

   Answer: RA $3^{\mathrm{h}}\,42^{\mathrm{m}}$ and dec $+26^\circ\,52'$. The point P' on the surface of any sphere that is furthest from a given point P on the surface of the sphere is the point obtained by following a diameter of the sphere from P through the center of the sphere until the diameter strikes the sphere again on the far side of the center at P'.

   If P is below the celestial equator by a declination $-26^\circ\,52'$, then P' is above the equator by the same amount so the dec of P' must be $+26^\circ\,52'$. One way to see this is to imagine a diameter whose ends are on the celestial equator and that can swing freely about the center of the sphere. Then like a see-saw, as one end of the diameter is pushed down to achieve some negative declination, the other end swings up from the celestial equator an equal angular distance.

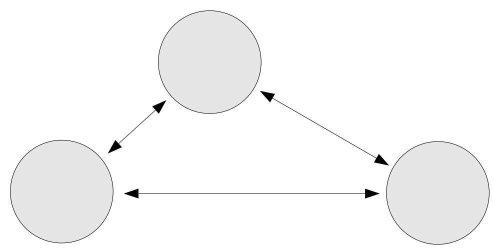
   Similarly, the point P' must have a right ascension 180° away from the RA of P. Again, you can see this by imagining a diameter whose ends are on the celestial equator starting at the spring equinox 0^h and ending at the fall equinox at 12^h. As you swing the diameter about its center to align one end with the given RA of $15^{\mathrm{h}}\,42^{\mathrm{m}}$, the other end is always 12^{h} away. So point P' has an RA of $15^{\mathrm{h}}\,42^{\mathrm{m}} - 12\mathrm{h} = 3^{\mathrm{h}}\,42^{\mathrm{m}}$. Alternatively, you can add 12 sidereal hours to get the RA of P': $15^{\mathrm{h}}\,42^{\mathrm{m}} + 12\mathrm{h} = 27^{\mathrm{h}}\,42^{\mathrm{m}} = 3^{\mathrm{h}}\,42^{\mathrm{m}}$ since 27 is equivalent to 3 when 24 hours takes you once around the celestial equator.

3. **(5 points)** Circle all objects in the following list that undergo retrograde motion as observed by someone on Earth. (No justification required here.)

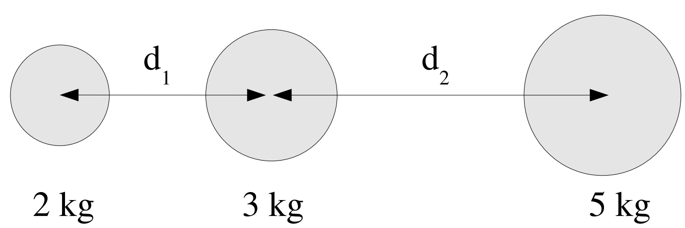
   (a) The Moon.
   (b) The Sun.
   (c) Venus.
   (d) Neptune.
   (e) The Andromeda galaxy.

   Answer: All planets undergo retrograde motion as observed by any other planet in our Solar System so Venus and Neptune are two immediate answers. (Fig. 2.32 on page 51 of the text actually holds for any two planets orbiting the Sun.) Since the Moon orbits the Earth and not the Sun, the Moon can not undergo retrograde motion since the Earth never "catches up to" or falls behind the motion of the Moon. Similarly, since the Earth orbits the Sun directly, the Sun does not undergo retrograde motion from the Earth. The Andromeda galaxy is so far away, about 2.5 million ly, that it is on the celestial sphere and moves rigidly with the celestial sphere, so no retrograde motion here either. (Alternatively, the Andromeda galaxy is so far away that we are not able to see it move against a background of stars that are even further away.)

4. (a) **(6 points)** Explain how to arrange three balls with masses of 2 kg, 3 kg, and 5 kg in outer space such that one of the balls will remain motionless when you release the balls at rest. Assume that no other masses are nearby (you are solving this problem in intergalactic space), and that the balls are not touching each other when released.



Answer: This problem is a simple variation of an example that I worked out in lecture, to illustrate how one has to add up various forces to get the total force on some given object, at which point you can determine the acceleration of the object from Newton's second law $\mathbf{a} = (1/m)\mathbf{F}_{\text{total}}$. A first observation is that the three balls have to be collinear (all on a line) to have one ball be stationary when released. If the balls are not collinear as in the above figure, the total force on each ball can not equal zero, e.g., you should be able to see that the middle ball above is being pulled down and to the left by the left ball, down and to the right by the right all, so the total force on the middle ball has a downward direction and the middle ball can't be stationary when released.



For three collinear balls, the order of the balls is completely arbitrary and one can find infinitely many different ways to arrange the balls so that the middle ball is stationary. (With three balls, only the middle ball can be stationary when released, the two outer balls have to move toward the middle ball when released.) If we arbitrarily put the 3 kg ball in the middle as shown above, we can also arbitrarily choose the distance d_1 between the 2- and 3-kg balls to be one meter (I recommend choosing the simplest possible numbers to work with) and see if we can adjust the other distance d_2 so that the total force on the middle ball is zero. The middle ball is pulled to the left by a gravitational force Gm_1m_2/d_1^2 and is pulled to the right by a force Gm_2m_3/d_2^2. The condition that the total force on the middle ball is zero is then:

$$\frac{Gm_1m_2}{d_1^2} = \frac{Gm_2m_3}{d_2^2} \qquad \text{or} \qquad d_2 = \sqrt{\frac{m_3}{m_1}}d_1. \tag{11}$$

Notice that if $m_3 > m_1$, then $d_2 > d_1$ which makes sense: the more massive ball m_3 has to be further away from the middle ball so that its force can be diminished enough by distance to balance the force from the closer smaller mass m_1.

If we substitute $d_1 = 1\,\mathrm{m}$, $m_1 = 2\,\mathrm{kg}$, and $m_3 = 5\,\mathrm{kg}$, we find that

$$d_2 = \sqrt{\frac{5}{2}}\,1\,\mathrm{m} \approx 1.58\,\mathrm{m}. \qquad (12)$$

For any order of balls along the line and for any choice of the distance d_1, we can choose a distance d_3 so there are infinitely many solutions.

(b) **(4 points)** For your arrangement of balls, determine and then indicate which ball feels the largest total force and which ball undergoes the largest acceleration.

Answer: The total forces on ball 1 and ball 3 respectively are

$$F_1 = m_1 a_1 = \frac{G m_1 m_2}{d_1^2} + \frac{G m_1 m_3}{(d_1 + d_2)^2} \quad \text{to the right}, \qquad (13)$$

$$F_3 = m_3 a_3 = \frac{G m_2 m_3}{d_2^2} + \frac{G m_1 m_3}{(d_1 + d_2)^2} \quad \text{to the left}. \qquad (14)$$

If you look at these expressions for a second or so, you will notice that the term $G m_1 m_2 / d_1^2$ in F_1 has the same value as the term $G m_2 m_3 / d_2^2$ in F_3 since we set these values equal to each other to make the middle ball have a zero net force. So we see that $F_1 = F_3$ and the total forces are equal in strength and opposite in direction for the two outer balls. *Thus no ball has a biggest total force, instead the two outer balls have the same nonzero total force.* The outer ball with the smallest mass then undergoes the largest acceleration, which here would be the 2 kg ball.

Note: In their answers, several students wrote Newton's law of gravity by adding the two masses rather than by multiplying them, for example they wrote

$$F_1 = \frac{G(m_1 + m_2)}{d_1^2}. \qquad (15)$$

This was so shocking I didn't know how many points to take off: this formula had been used in lecture, everyone had done some homework problems using this formula, and poor Isaac must have turned over in his grave. While it may seem that adding the masses is no different mathematically or logically than multiplying them, it makes a profound difference in terms of how nature actually works since an additive form of Newton's law would no longer allow objects with different masses to fall with the same acceleration to Earth:

$$m_1 a_1 = \frac{G(m_1 + m_2)}{d^2} \quad \Rightarrow \quad a_1 = \frac{G(1 + m_2/m_1)}{d^2}, \qquad (16)$$

and the acceleration a_1 would now depend on the mass m_1 of the object via the term $1 + m_2/m_1$, contrary to experiment. So the multiplicative form of Newton's law is crucial so that masses divide out and Galileo's law of different masses falling with the same acceleration will hold as observed.

Also, many students when working this problem out forgot to square the distance in the denominator. This is a common mistake during a test, especially when using a calculator to evaluate some combination of many numbers. I did not take off credit but do stay alert for such mistakes.

5. **(8 points)** A strong wind over many days has unfortunately blown your hot-air balloon far off course. Fortunately, you have some astronomical tables and instruments including a UT clock. With some effort, you determine the following information:

- You are able to identify the brightest star in your sky as Sirius.
- You observe Sirius on the southern part of your meridian with an altitude of 50°.
- According to your watch, Sirius is on your meridian at 11 pm.

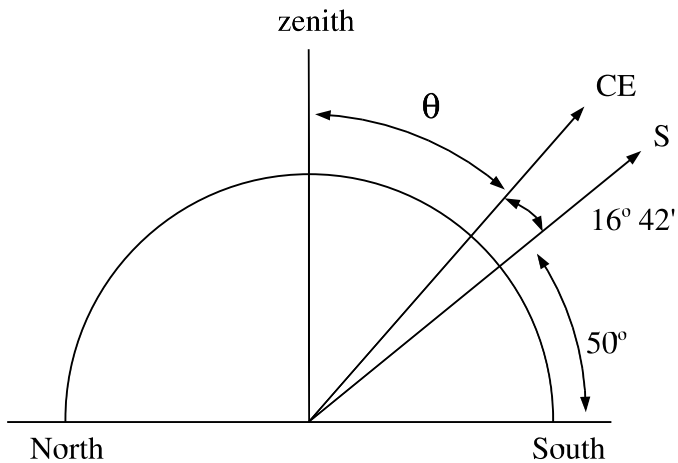
- At the same time, you observe that your UT clock reads 1 pm (13:00).
- Your astronomical tables indicate that Sirius has celestial coordinates RA $6^h\,45^m$, dec $-16°42'$ for epoch time 2000.

Determine your longitude and latitude (the latter in degrees and arcminutes). Justify your answer with an appropriate drawing.

Answer: One part of this problem was unfortunately not worded correctly and caused some confusion, the statement "you observe that your UT clock reads 1 pm (13:00)" should have read "according to a table and to your UT clock, Sirius crosses the meridian of Greenwich, England, at 1 pm (13:00) UT time."

The longitude part of this problem should have been straightforward. Your local watch reads 11 pm which is 23:00 on a 24-hour clock, while the UT clock reads for the same event (a certain star crossing the meridian) the time 13:00. Your local time is therefore 23:00 - 13:00 = 10 hours ahead of Greenwich time, which means that your longitude is $10\,\mathrm{h} \times 15°/\mathrm{h} = 150°$ degrees *east* of the prime meridian.

Many students incorrectly tried to use the right ascension of Sirius to determine longitude, e.g., some students subtracted the RA of $6^h\,45^m$ from the local time, but this doesn't make sense, you can't combine celestial coordinates with earth coordinates. Some other students were not careful to note that the local time was 11 pm in the evening (if you can see stars on your meridian it is most definitely night time), and subtracted 11 am from the UT time of 1 pm to get a time difference of 2 hours and a longitude of 30° W, which would represent a huge error in your location.



Your latitude could be determined by figure similar to the above. You know that the star Sirius (the letter S in the figure) has an altitude of 50° on the southern side of your meridian as shown. We also are told that Sirius has a declination of $-16°42'$, which means that Sirius is below the celestial equator CE and so closer to the south celestial pole. So given the position of Sirius on the meridian, the celestial equator must be an angular distance $16°42'$ closer than Sirius to the north celestial pole as shown. Thus the celestial equator must have an altitude of $50° + 16°\,42' = 66°\,42'$ up from the southern horizon. We learned in the course that the latitude is the altitude of the star Polaris *or* the angle between the zenith and the celestial equator. So the latitude is given by

$$90° - (66°\,42') = (89° - 66°) + (60' - 42') = 23°\,18'\,\mathrm{N}, \tag{17}$$

where the latitude is north since the celestial equator is on the southern part of the meridian.

You can do *one* of the following optional problems to get extra credit.

1. **(6 points)** President Bush has proposed to put a base on the Moon from which future flights to Mars will be launched. By appropriate calculations, determine the minimum launch speed of a rocket (minimum to save money) from the Moon base that will allow the rocket to escape from both the Moon and Earth. In answering this problem, you will need to determine and state where on the Moon the rocket should be launched to achieve this minimum launch speed. Based on your calculations, state whether there is an economic advantage in launching rockets to Mars from the Moon instead of from Earth.

   Answer: The formula $v = (2GM/r)^{1/2}$ for the escape speed v of some object (think rocket) that is a distance r from the center of mass of some other object of mass M (think Earth or Moon) was obtained by equating the kinetic energy of the rocket $(1/2)mv^2$ to the negative of the gravitational potential energy GmM/r at the point where the rocket is being launched. The speed v is the initial speed of the rocket, and far away from the mass M, the speed of the rocket will have dimished to zero since all of its kinetic energy will have converted to potential energy. (The sitution is similar to the motion of a person on a swing: at the bottom of the swing, the speed is maximal and the potential energy is minimal, while at the peak of the swing, the speed is zero and the potential energy is maximal.)

   If there are several masses present, the total gravitational potential energy at a given point is the sum of the gravitational potential energies GmM/d arising from each mass separately at that point. For a rocket being launched in the vicinity of the Earth and Moon, the minimum speed to leave the Earth-Moon system, (i.e., to leave the combined masses of Earth and Moon) would be

   $$\frac{1}{2}mv^2 = \frac{GmM_{\text{Earth}}}{d_1} + \frac{GmM_{\text{Moon}}}{d_2}, \qquad (18)$$

   where d_1 is the distance of the rocket to the Earth's center of mass, and d_2 is the distance of the rocket to the Moon's center of mass. Since the mass m of the rocket divides out from both sides of this equation, the escape speed again does not depend on the mass of the rocket.

   There is a subtlety here: since the escape speed in Eq. (18) depends on the distances d_1 and d_2 of the masses to the point in space from which a rocket will try to escape, Eq. (18) makes sense only if these masses are frozen in space at fixed positions, i.e., if we can ignore the orbital motion of the Moon about the Earth and can ignore the orbital motion of the Earth about the Sun. Otherwise, as a rocket starts to leave the vicinity of the Moon, the total potential energy at any point in space varies with time (the distances of the launch point to the Earth and Moon change), leading to a difficult mathematical problem. So an escape problem from two or more masses in general can not be solved in any simple mathematical way if the locations of the masses change substantially during the time that it takes the rocket to escape.

   To get some intuition about how launching from the Moon may or may not be better than launching from the Earth, let's make the unrealistic assumption that the Moon and Earth indeed have fixed locations in space the entire time that the rocket follows its escape route, in which case we can use Eq. (18). If the rocket is launched from the Moon's surface (why to launch from the Moon's surface is already not obvious, one could imagine launching from either Earth orbit or a Moon orbit or from some other point in space like a stable Lagrange point), the distance d_2 would be the radius of the Moon R_{Moon}. We can make the launch speed in Eq. (18) smaller by making d_1 bigger, which would mean the rocket base should be on the far side of the Moon, as far from Earth's center of mass as possible. However, since the Moon is about 30 Earth diameters away and the Moon's radius is about one third of the Earth's radius, locating the base on the far side of the Moon gives only a tiny improvement, less than a percent.

   To escape from the Earth and Moon from a location on the far side of the Moon's surface, the desired speed v would be

$$v = \sqrt{2G\left(\frac{M_{\rm Earth}}{d_1} + \frac{M_{\rm Moon}}{d_2}\right)} = \sqrt{2G\left(\frac{M_{\rm Earth}}{d_{E-M} + R_{\rm Moon}} + \frac{M_{\rm Moon}}{R_{\rm Moon}}\right)} \quad (19)$$

$$\approx \sqrt{2(7\times 10^{-11})\left(\frac{6.0\times 10^{24}\,{\rm kg}}{(3.84+0.017)\times 10^{8}\,{\rm m}} + \frac{7.3\times 10^{22}\,{\rm kg}}{1.7\times 10^{6}\,{\rm m}}\right)} \quad (20)$$

$$\approx 2.9\,{\rm km/s}, \quad (21)$$

which is just a little bigger than the escape speed from the surface of the Moon

$$v_{\rm Moon} = \left(\frac{2GM_{\rm Moon}}{R_{\rm Moon}}\right)^{1/2} \approx \left(\frac{2\times(7\times 10^{-11})\times(7.3\times 10^{22})}{1.7\times 10^{6}\,{\rm m}}\right)^{1/2} \approx 2.4\,{\rm km/s}. \quad (22)$$

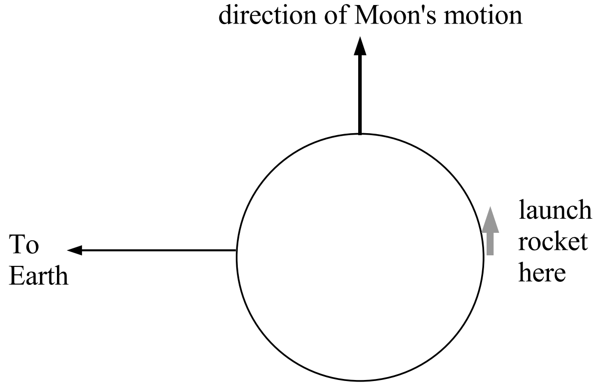
These values should be compared with the escape speed of a rocket from the surface of the Earth (ignoring the Moon), which you can show to be $(2GM_{\rm Earth}/R_{\rm Earth})^{1/2} \approx 11\,{\rm km/s}$, and the escape speed of a rocket from the Earth but at the same distance as the Moon from the Earth (again ignoring the Moon), $(2GM_{\rm Earth}/d_{\rm E-M})^{1/2} \approx 1.5\,{\rm km/s}$.

Launching from the surface of the Earth requires a much greater speed, 11 km/s, than launching from the Moon's surface, 2.9 km/s, or from a point not close to the Moon but the same distance as the Moon, 1.5 km/s. You might think that if you were to launch from the Moon's surface with a speed of 2.4 km/s just to escape the Moon, that you would automatically escape from the Earth since the escape speed from the Earth at the distance of the Moon is 1.5 km/s, but this does not take into account how the potential energies combine via Eq. (18). Provided one could amortize the enormous expense of building a Moon base over many decades by launching many rockets from the Moon (this does not seem likely to me), then it may be more economical to launch rockets from the Moon than Earth because of the substantially lower escape speeds. Would be a good extra credit project for someone to try to nail down these details and compare the scientific and economic advantages of launching from Earth, from the Moon, or from some place in space.

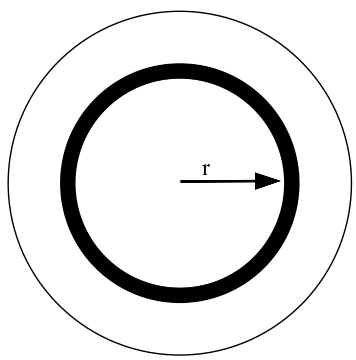
What is the smallest speed with which one can launch a rocket from the Moon and achieve escape velocity from the Moon and Earth? Again, the problem is too hard to solve exactly if you allow the Moon and Earth to move about during the time that the rocket is in motion, but another insight can be obtained by taking into account the motion of the Moon, while assuming somewhat inconsistently that the distances d_1 and d_2 in Eq. (18) do not change. All students who tried this extra credit problem missed the implication that a rocket on the Moon is already in motion because the Moon is itself in motion about the Earth (more precisely, about the Earth-Moon center of mass). The speed of the Moon in its approximately circular orbit around Earth is

$$v_{\rm Moon} = \frac{2\pi d_{\rm E-M}}{p_{\rm Moon}} \approx \frac{2\pi(3.8\times 10^{5}\,{\rm km})}{27.3\,{\rm d}\times 24\,{\rm h/d}\times 3600\,{\rm s/h}} \approx 1.0\,{\rm km/s}, \quad (23)$$

which is not small compared to the escape speed of 2.9 km/s. *So one could obtain a decrease in the launch speed by firing the rocket in a horizontal position from the Moon's equator on the far side of the Moon, with a speed of 2.4 km/s relative to the Moon, in the direction of the Moon's orbit about the Earth (see figure below).* The velocity of the rocket will then add to the velocity of the Moon to give a total escape speed of 2.4 + 1.0 = 3.4 km/s, in excess of the Moon-Earth escape speed. In particular, because of the Moon's motion, one would not have to launch a rocket with the Moon-Earth escape speed of 2.9 km/s. Again, this discussion works only to the extent that we can assume that the Earth and Moon have fixed locations during the escape of the rocket.



For a similar reason, most rocket bases on Earth are located as close to the equator as possible, since rockets launched at the equator get a boost to their launch speed from the rotational speed of the Earth, which is fastest at the equator, about 0.5 km/s compared to the Earth's escape speed of 11 km/s. However, rockets are not launched horizontally on Earth (although this would work) since at high speeds it is more expensive to fly a rocket through the greater length of atmosphere (greater friction through the air) compared to launching vertically.



2. **(6 points)** The Moon is basically a big rock with constant mass density of $\rho \approx 3,000\,\mathrm{kg/m^3}$. To facilitate transport around the Moon, a giant hollow circular tube of radius r is carved out of the Moon's interior as shown in the figure above (the tube is the thick black circle). Derive a mathematical expression for the period of a subway car to orbit once around inside this hollow tube as a function of distance r from the Moon's center, assuming the interior of the tube is in vacuum. Estimate the round-trip time in minutes for $r = 1,400\,\mathrm{km}$, a bit less than the Moon's radius $r \approx 1,700\,\mathrm{km}$, and state your opinion whether this transportation system would be fast enough to be useful.

   Answer: This is a problem analogous to the discovery of dark matter using rotation curves: the circular orbit of a subway car in such a tunnel involves the orbital velocity law $M_r = rv^2/G$, where M_r is the total mass enclosed within a circular orbit of radius r. If the tunnel has a radius r and the density of the Moon is assumed to be constant with value ρ (lower case Greek rho), then the total mass inside the tunnel is

$$M_r = \text{“mass density”} \times \text{“volume”} = \rho \left(\frac{4}{3} \pi r^3 \right). \tag{24}$$

   The speed v of subway car's orbital motion is related to the radius of its orbit and its orbital period p by

$$v = \frac{2\pi r}{p}. \tag{25}$$

Substituting this last expression for v in terms of p into the orbital velocity law to eliminate v, substituting the expression for M_r in terms of ρ and r into the orbital velocity law, and simplifying, one discovers that all powers of the radius cancel so *the orbital period p is independent of the radius of the tunnel*:

$$p = \left(\frac{3\pi}{G\rho_0} \right)^{1/2} , \tag{26}$$

which is the desired formula. Substituting numerical values gives an orbital period of

$$p \approx \left(\frac{3 \times 3}{(7 \times 10^{-11}) \times (3 \times 10^3)} \right)^{1/2} \approx 6{,}500 \text{ s} \approx 1.8 \, \text{hr}. \tag{27}$$

This is quite a fast time to orbit completely around the Moon and it is interesting that the time does not depend on radius. But building such a tunnel makes no sense since one obtains the same period just above the surface of the Moon, so one might as well just rocket from one part of the Moon to another. An exception might be if one wanted to live mainly underground to avoid solar radiation and damage from meteors.

This problem gives some insight about the structure of galactic rotation curves. If the mass near the center of a galaxy has a constant density ρ, then the orbital velocity law becomes

$$M_r = \rho \times \left(\frac{4}{3} \pi r^3 \right) = \frac{r v^2}{G} \qquad \Rightarrow \qquad v = \left(\frac{4\pi\rho G}{3} \right)^{1/2} r, \tag{28}$$

i.e., the orbital speed increases linearly with the radius, and by measuring the slope of the line from an empirical rotation curve and by equating the estimated slope to the mathematical exression for the slope $(4\pi\rho G/3)^{1/2}$, we can deduce the mass density ρ. So the rapid approximately linear increase in v with r in Figure 19.19c on page 615 of the text can be understood by assuming that the density of mass is approximately constant out to about 7,000 ly. (Why the density is approximately constant is another thing, it is not obvious since clouds of matter contract under gravitation and tend to have an increasing density closer to the center of the cloud.) For larger distances, the density starts to decrease with distance, but does not vanish at the visible edge of the galaxy.