1 Introduction

Before the discovery of the neutrino oscillations, there was no evidence for neutrino mass, and neutrinos were commonly treated as massless particles to account for the lack of observation of right-handed neutrinos. The massive nature of the neutrinos brings many questions including, why the neutrino masses are much smaller than other particles, and why there are no right-handed neutrinos.

Models that can answer these questions also predict non-standard neutrino interactions. The determination of absolute neutrino masses is critical because most predicted non-standard neutrino interactions, depend on a combination of these masses (For example Majorana mass responsible from the neutrinoless double beta decay). Lack of knowledge on the absolute neutrino masses prevents elimination of certain theories despite null observations, because these theories could still be correct for lower neutrino masses. Therefore a model independent measurement of absolute neutrino mass is necessary in the case of continuing null observations of these interactions. One model independent way to determine neutrino masses is inspecting the electron energy spectrum of tritium beta decay.

In this paper first, neutrino oscillations and hierarchy problem will be briefly described. Then the general principle of tritium beta decay experiments, and how individual neutrino mass eigenvalues can be found with the combination of oscillation data and tritium beta decay experiments will be discussed. Finally, one current and one next generation tritium beta decay experiment, namely KATRIN and Project 8, will be described.

2 Neutrino Oscillations

Neutrino oscillations arise because neutrino flavor and mass eigenstates are different. That is to say a flavor eigenstate do not have a specific mass, but is a superposition of mass eigenstates which have definite mass eigenvalues. The unitary matrix stating the mixing between neutrino flavor and mass eigenstates can be parametrized as in Equation 1 [1] assuming neutrinos are Dirac fermions1.

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1If neutrinos are Majorana fermions, two additional phases called the Majorana phases will be introduced.
probabilty which can be seen in Equation 2 reects these results mathematically. The functional form of three a v or neutrino oscillation will be greater and the change in the a v or eigenstate will be faster. Of course if the neutrino energy is very different a v or of neutrino. Clearly, this oscillation probabilit y will depend on the dierence between phases. Since a a v or eigenstate is a sup erp osition of dierent mass eigenstates, in time each will obtain a according to the quan tum mec hanical time ev olution, meaning each energy eigenstate

\[ \nu_\alpha = U_{\alpha 1} \nu_1 + U_{\alpha 2} \nu_2 + U_{\alpha 3} \nu_3 \]

In Equation 1, \( c_{ij} = \cos(\theta_{ij}) \), \( s_{ij} = \sin(\theta_{ij}) \), where mixing angles \( \theta_{ij} = [0, \pi/2] \) and CP violating phase \( \delta = [0, 2\pi] \) have the specied ranges. In Equation 1 and in the following equations greek indices represent flavor eigenstates, while the number indices represent the mass eigenstates of neutrinos. For the anti-neutrinos the mixing matrix would be the complex conjugate of this one.

Neutrinos are created in deinite flavor eigenstates. However, the wavefunctions of these neutrinos evolve according to the quantum mechanical time evolution, meaning each energy eigenstate\(^2\) will obtain a different phase. Since a flavor eigenstate is a superposition of different mass eigenstates, in time each will obtain a different phase, and neutrino will not be in the same flavor eigenstate anymore, and can possibly interact as a different flavor of neutrino. Clearly, this oscillation probability will depend on the dierence between phases obtained, hence if the mass splitting between different mass eigenstates are greater, the phase dierence will be greater and the change in the flavor eigenstate will be faster. Of course if the neutrino energy is very high, then the small mass dierences between mass eigenstates would not be so important, and oscillation frequency in the same conditions would be lower. The functional form of three flavor neutrino oscillation probability which can be seen in Equation 2 reects these results mathematically.

\[ P_{\alpha \to \beta} = \delta_{\alpha \beta} - 4 \sum_{i>j} Re\{U_{\alpha i}^* U_{\beta j} U_{\alpha j} U_{\beta i}^*\} \sin^2\left(\frac{\Delta m_{ij}^2 L}{2E}\right) + 2 \sum_{i>j} Im\{U_{\alpha i}^* U_{\beta j} U_{\alpha j} U_{\beta i}^*\} \sin\left(\frac{\Delta m_{ij}^2 L}{2E}\right) \]

In Equation 2, \( \Delta m_{ij}^2 = m_i^2 - m_j^2 \), \( E \) is energy and \( L \) is distance traveled\(^3\). One important conclusion of this expression is that since the rst two terms are independent of the sign, if the contribution of the last term in the expression is less than the experimental uncertainties, then the sign of \( \Delta m_{ij}^2 \) cannot be determined by that experiment. This leads to the neutrino mass hierarchy problem.

As previously mentioned, current oscillation experiments are not sensitive to the sign of mass dierences, when only vacuum oscillations are considered. However, when neutrinos travel in matter, the oscillation probabilities are modied due to the possible neutrino interaction. The oscillation probability can still be given in the form of equation 2, however mixing angles and mass squared dierences will be modied. By inspecting how this modication affects oscillation probabilities, it is possible to obtain the sign of \( \Delta m_{ij}^2 = m_i^2 - m_j^2 \). In fact that is how we know \( \Delta m_{21}^2 > 0 \) [2], since solar neutrino oscillations are due to this splitting and sun provides a long distance in matter to study matter eect.

However, the sign of \( \Delta m_{31}^2 \) is still unknown, and hence the hierarchy of the neutrino masses. As seen in the Figure 1, \( m_3 \) could either be greater than \( m_1 \) and \( m_3 \) (normal hierarchy), or the other way around (inverted hierarchy).

Since oscillation probability only depends on the squared mass dierences, these experiments could not

\(^2\) Since all mass eigenstates share the same momentum, the dierence in dierent energy is only due to mass and mass eigenstates are also energy eigenstates.

\(^3\) In natural units.
measure the absolute mass scale of the neutrinos. However, these experiments set a lower limit to the neutrino masses and give the relationship between different neutrino mass eigenstates. Therefore, measurement of any combination of neutrino masses, in addition to the resolution of neutrino hierarchy is enough to determine all three neutrino mass eigenvalues. One promising way to measure absolute neutrino mass is the tritium beta decay experiments, also called the tritium endpoint method.

3 Determining the Neutrino Mass with Kinematic Measurements

3.1 Tritium Endpoint Method

Tritium endpoint method, uses the beta decay of tritium to measure or upper bound the neutrino mass. These experiments measure the high end of the beta decay electron spectrum, to measure how much it deviates from a spectrum of zero neutrino mass. Basically, the electron spectrum will end sharply at a lower energy than it would in the case of zero neutrino mass, as depicted in Figure 2.

The neutrino mass\(^4\) measured with this method is called electronic neutrino mass (or beta decay mass). This mass is the square root of the expected mass square of the electron neutrino flavor eigenstate\(^5\), mathematically given in equation 3.

\[
m_{\text{electronic}} = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2} \quad (3)
\]

Measured oscillation parameters allow us to parametrize equation 3 with respect to the lightest neutrino mass as in equations 4 and 5, respectively for normal and inverted hierarchy. When the current mixing and mass difference values were inserted and the relation graphed, Figure 3 [3] could be obtained.

\[
m_{\text{electronic}} = \sqrt{c_{12}^2 c_{13}^2 m_{\text{lightest}}^2 + s_{12}^2 s_{13}^2 (m_{\text{lightest}}^2 + \Delta m_{21}^2) + s_{13}^2 (m_{\text{lightest}}^2 + \Delta m_{31}^2)} \quad (4)
\]

\(^4\)Actually from beta decay, anti-neutrinos are created, but since their mass should be same, it will be called neutrino mass from now on.

\(^5\)The expectation is over mass squared instead of mass because tritium endpoint method is directly measuring square of the mass.
Figure 2: On the left, normalized tritium beta decay spectrum. On the right, the endpoint of the spectrum is zoomed in, and the shape of spectrum with 0 and 1 ev neutrino beta decay masses were plotted. Only a fraction of $2 \times 10^{-13}$ of all the decays occur at the region of interest of the spectrum [4].

\[
m_{\text{electronic}} = \sqrt{c_{12}^2c_{13}^2(m_{\text{lightest}}^2 - \Delta m_{31}^2)} + s_{12}^2s_{13}^2(m_{\text{lightest}}^2 + \Delta m_{21}^2 - \Delta m_{31}^2) + s_{13}^2m_{\text{lightest}}^2
\]

(5)

Figure 3: Electronic Neutrino Mass (also known as Beta Decay Mass) versus Lightest Neutrino Mass [3]. The constraints imposed by the oscillation experiments can be seen in the red (normal hierarchy) and in blue (for inverted hierarchy) intervals determined by the same color solid lines. The current limit of the PLANCK experiment and expected limits of KATRIN and Project 8 experiments are shown in green dashed lines.
Red and blue curves in the Figure 3 neatly summarizes our knowledge about absolute neutrino mass coming from the oscillation experiments. Dashed green lines show projected limits from current (KATRIN) and next generation (Project 8) tritium beta decay experiments and current limit from the cosmological observations (Planck). It can be seen that if electronic neutrino mass is measured above 40 meV, the neutrino mass hierarchy still should be resolved by other means to obtain the absolute neutrino masses. However, if electronic neutrino mass is limited below by tritium beta decay experiments, then hierarchy problem will be resolved automatically in the favor of normal hierarchy.

The principle of tritium beta decay experiments are simple and model independent, but the experiment itself is challenging. First of all, the energy spectrum should be measured with an energy resolution on the order of 1 eV or less. At the tritium endpoint, this corresponds to 1 eV in 18.6 keV, or a resolution of 0.005% [3]. Second, only 1 decay in a total of $5 \times 10^{14}$ decays have the electron energy at the last 1 eV of the spectrum. Therefore large sources of tritium and long operation times are necessary to obtain good statistics. In addition there are systematic uncertainties related to the final state energy of the decaying tritium, which also shares the energy available in the beta decay.

The current best limit for the electronic neutrino mass, from the tritium beta decay measurements are set by the Troitsk Experiment [5], as 2.05 eV. Currently cosmological limits to the neutrino mass [6] are one order of magnitude ahead of limits set by tritium beta decay experiments.

### 3.2 KATRIN Experiment

KATRIN Experiment is actually the scaled version of the previous tritium beta decay experiments Troitsk and Mainz, which were using a spectrometer called MAC-E Filter [7] (Magnetic Adiabatic Collimation with Electrostatic Filter). As can be seen in Figure 7, by taking data until 2022, KATRIN Experiment will either measure the electronic neutrino mass or limit it below 0.2 eV. The main spectrometer of KATRIN and it’s operation principle can be seen in Figure 4. The negative potential at the center slows down incoming electrons from the source, allowing only the ones with more kinetic energy than $qU_{\text{max}}$ to pass, reflecting lower energy ones to the source. Therefore, main spectrometer works as a high pass filter, measuring the integral flux of electrons over a certain energy. The potential is swept so that electron counting at different energy thresholds can be made, and hence the electron energy spectrum is obtained.

However, such a system would not be able to account for transverse momentum of the electrons, and hence measure the electron energy inaccurately, if not for the magnetic adiabatic collimation. Due to superconducting solenoids on both sides of the spectrometer (at the source and at the detector side), there is a magnetic field along the spectrometer axis. The magnetic field is strongest in the source and detector regions, and lowest at the center of the spectrometer, however the change is continuous and slow (hence the word adiabatic). Under an adiabatically changing magnetic field, the magnetic moment of a rotating charge stays constant and given by the Equation 6. Since B decreases at the center of the spectrometer, to keep the magnetic moment, transverse momentum of the electron should also decrease. Magnetic force cannot do work on the particle, hence the transverse momentum must be converted into parallel momentum with this mechanism.

$$\mu = \frac{p_\perp^2}{2mB}$$

(6)

Since, the amount of transverse energy transferred to longitudinal energy is determined by the ratio of the minimum magnetic field to the maximum magnetic field, the energy resolution of the detector is proportional to this ratio [7] as in Equation 7. This ratio gives a resolution of 0.93 eV at the tritium endpoint energy of 18.6 keV.

$$\Delta E = \frac{B_{\text{min}}}{B_{\text{max}}} \times E_e$$

(7)

The 23 meters long main spectrometer expands in the center up to 9.8 m in diameter [7], in order to give electrons space to move until their transverse momentum is converted to parallel momentum. In order to
increasing statistics with this method, the only way is to increase the radial size of the source \(^6\) [3], resulting in a need for a proportional increase in the spectrometer size. KATRIN will probably be the last tritium beta decay experiment, due to the massive cost of scaling such a system one order of magnitude more.

### 3.3 Project 8 Experiment

Project 8 is a next generation tritium beta decay experiment, aiming to reach a sensitivity of 40 meV that can be seen in Figure 3, by using cyclotron radiation emission spectroscopy (CRES) [3] instead of a MAC-E filter. This method determines the energy spectrum of the beta decay electrons by measuring the frequency of photons emitted during the cyclotron motion of electrons within a magnetic field. The characteristic frequency of the cyclotron motion and hence the emitted photons are given as in the Equation 8.

\[
\omega = \frac{eB}{\gamma m_e}
\]  

As seen in Equation 8, the frequency of the measured photon would be inversely proportional to the energy of the electron, hence accuracy of the frequency measurement is directly reflected to the energy measurement, only affected by the uncertainty in the magnetic field \(B\). Of course, such a method would not be useful if the power of the radiation emitted by a single electron proves to be too low to be detected. Phase 1 of the Project 8 is dedicated to showing that cyclotron radiation from a single electron trapped in a magnetic field can be accurately measured [3]. In Figure 5 a Short Time Fourier Transform (STFT) of the measured signal originated from a single electron can be seen. Although the power of radiation emitted from a single electron is in the order of \(-120\, dBm = 10^{-15}\, Watts\), the signal due to cyclotron radiation is clearly visible over the background. The increase in the cyclotron frequency with time (with the energy loss of the electron) can also be observed in Figure 5. Although CRES was never utilized before, it has multiple advantages [8] over the MAC-E filter method used in previous and current tritium beta decay experiments:

\(^6\)Increasing the density and length of the source makes electronic interactions within the Tritium non-negligible.
Figure 5: Detection of the cyclotron frequency emitted by a single electron with an STFT plot [3]. Notice that frequency is slightly increasing as the electron continues to lose energy.

**Source = Detector:** In experiments using MAC-E filter method, one needs to transport electrons from source to detector, through the spectrometer. Therefore, electrons not emitted in the detector direction could not be sampled. In addition, since electrons must reach the detector to be counted, the thickness or density of the source could not be increased without also increasing the scattering probability of electrons within the source. Since electrons do not need to reach a detector with the CRES method and microwave photons do not interact with tritium, these two limitations do not apply to Project 8, as long as electrons can travel several microseconds to emit cyclotron radiation.

**Frequency Measurement:** Frequency is one of the most accurately measured quantities with current detector technology. The frequency range of emitted photons are in the microwave range, where with enough sampling time it is possible to precisely measure the frequency, resulting in optimum energy resolution.

**Full Spectrum Sampling:** When KATRIN is measuring the high end of the energy spectrum, electrons with lower energies are reflected back to the source. This process greatly lowers the statistical efficiency of traditional spectroscopic methods. With CRES method all the photons emitted by the electrons can be measured simultaneously, resulting in a much higher statistical efficiency.

Previously, improvements offered by Project 8 on statistics and energy resolution were mentioned. Project 8 also aims to lower systematic uncertainties when compared to KATRIN by using an atomic tritium source [3] instead of a molecular tritium source. In Figure 6 sensitivities that can be achieved with different densities of tritium sources (atomic and molecular) as a function of effective volume can be seen. The worst sensitivity
is obtained when molecular tritium with density $3 \times 10^{13}$ cm$^{-3}$ is used because electrons interact with the tritium molecules before enough CRES photons are detected. This problem can easily be avoided by choosing a lower density tritium source with an equal effective volume. At lower densities of molecular tritium, the limiting case would be systematic uncertainties regarding final state of decayed tritium molecules. When an atomic tritium source was utilized the systematic uncertainties due to the final state spectrum of the tritium are mostly evaded, and a sensitivity of 40 meV to electronic neutrino mass could be obtained with a large enough detector.

![Figure 6](image)

Figure 6: The sensitivity of the Project 8 experiment with different concentrations of molecular (blue) or atomic tritium (red) as a function of effective volume [3].

Although, Project 8 offers improvements on every side, there are still major engineering challenges for the experiment. Currently, the photons emitted during the cyclotron motion was collected via a single waveguide. In order to have a relatively large source and be able to detect photons emitted in all directions, the detector must be designed as an antenna array [3]. In addition, molecular and atomic tritium have different beta decay spectrum endpoint energies. Therefore, the purity of atomic tritium source should be kept better than one part in a million,[3] in order to prevent having background from beta decay spectra of molecular tritium. The expected development timeline of Project 8, with respect to KATRIN and cosmological experiments can be seen in Figure 7.

### 4 Conclusion

Although neutrino oscillations showed that neutrinos have mass, these experiments cannot measure the absolute mass of the neutrinos. By inspecting the kinematic restrictions imposed on the beta decay electron spectrum by the neutrino mass, tritium beta decay experiments are offering a model independent way to measure electronic neutrino mass. It is shown that with this measurement and the resolution of the neutrino hierarchy, three neutrino mass eigenvalues will be determined. Next generation tritium beta decay experiments have the potential to measure electronic neutrino mass down to 40 meV, and in the case of null observation the hierarchy would be resolved by these experiments.
Figure 7: A timetable for the future mass limits [3]. Notice that cosmological observations currently have stricter limits than terrestrial tritium beta decay experiments.

It should also be noted that although being model dependent, cosmological observations [6] and neutrinoless double beta decay experiments [9] have much stricter limits on the neutrino mass. While it is highly possible that first measurement of neutrino mass comes from these type of experiments, a model independent confirmation would still be necessary, and in the case that these experiments give null results while tritium beta decay experiments measure the neutrino mass, these models would be excluded.

References


