Carbon nanotube quantum dots: SU(4) Kondo and the Mixed Valence regimes

Gleb Finkelstein

Duke University

Alex Makarovski

Thanks:

Support: NSF DMR-0239748
Electronic band structure, sample
Coulomb blockade
Kondo effect / Mixed Valence
-apparent Kondo behavior in the Mixed Valence regime
Samples

\[ \text{V}_{\text{source-drain}} \quad \text{V}_{\text{gate}} \]

A

nanotube

SiO\textsubscript{2}

doped Si

Measurement: differential conductance
\[ = \frac{\text{d}I}{\text{d}V} (\text{V}_{\text{gate}}) \]

Single-wall CNT \( \sim 2 \) nm in diameter
Graphene (2D graphite): semi-metal

Energy dispersion $E(k)$

"Physical Properties of Carbon Nanotubes"
Saito, Dresselhaus and Dresselhaus 1998.
Nanotubes: quantization of quasi-momentum

\[ k_\perp \]

\[ k_\parallel \]

\[ C = 2\pi R \]

\[ r + C \text{ coincides with } r \Rightarrow k_\perp = 2\pi N/C = N/R \]

\[ k_\parallel \text{ - arbitrary} \]
Nanotubes: metallic and semiconducting

- Metallic or semiconducting
- Two degenerate bands

k- quasi-momentum along the length of the nanotube
Degenerate orbitals: ‘shells’

Quantization along length

Two degenerate subbands

=> degenerate orbitals

W.J. Liang, M. Bockrath, and H. Park, PRL (2002)

M. R. Buitelaar et al., PRL (2002)

shell
Electronic band structure, sample

Coulomb blockade

Kondo effect / Mixed Valence

-apparent Kondo behavior in the Mixed Valence regime
Ambipolar semiconducting nanotube

max: $4e^2/h$

$T=1.5K$

Conductance ($e^2/h$)

Vgate (Volts)

band gap

P-type

N-type

holes

electrons
4 degenerate levels in the dot are coupled one-to-one to 4 modes (↑, ↓, ↑, ↓) in the leads. The modes are not mixed by tunneling. Tunneling Hamiltonian has SU(4) symmetry.

M.S. Choi, R. Lopez and R. Aguado, PRL (2005)
Tunneling grows with Vgate

$\Gamma$ grows

Energy gap

Conductance ($e^2/h$)
Groups of 4 peaks: orbital degeneracy

4 degenerate states:
2 degenerate orbitals x 2 spin projections

“shell”
Groups of 4 peaks: orbital degeneracy

\[ E \]

\[ k_0 \]

\[ \Delta E \] – QM level spacing

\[ E_C = \frac{e^2}{C} \] – Charging energy
Conductance map: $G(V_{gate}, V_{sd})$
Nanotubes in Magnetic field

Orbital + Zeeman

Zeeman

Ando, SST (2000)
Nanotube in $B||$

Orbital + Zeeman

$V_{\text{gate}}$ (V)

Magnetic field (Tesla)

conductance
$B_\perp$: Zeeman splitting

\[ S = 0, 1 \]

\[ S_z = 1 \]
Addition energies in $B_\perp$

Addition energy = peak spacing in $V_{\text{gate}}$ converted to energy
Perpendicular magnetic field: Zeeman splitting

Addition energy

Exchange

Exchange is negligible

Electronic band structure, sample
Coulomb blockade
Kondo effect / Mixed Valence
-apparent Kondo behavior in the Mixed Valence regime
Kondo effect in Quantum Dots

Conductance grows at low temperatures in valleys with a non-degenerate ground state

Theory:
- Glazman and Raikh (1988)
- Ng and Lee (1988)

Experiment:
- Cronenwett et al., (1998)
Kondo effect in Nanotubes

SU(4) theories for 1 electron:
- Double dots, dots with symmetries:
  - D. Boese et al., PRB (2002)
  - G. Zarand et al., SSC (2003)

Nanotubes:
- M.S. Choi et al., PRL (2005)

1 electron SU(4) Kondo experiment:
- Quantum dots:

Nanotubes:
Kondo effect in Nanotubes

Interactions \( \uparrow \downarrow = \uparrow \downarrow = \uparrow \downarrow \)
Exchange \( \uparrow \uparrow \) is small

2 electron SU(4) theory

2-e Kondo in nanotubes – triplet or SU(4) ?
W.J. Liang, M. Bockrath and H. Park, PRL (2002)
Kondo effect in Nanotubes

Friedel sum rule: $\sum \delta_i = \pi N$
Kondo singlet: $\delta^\uparrow = \delta^\downarrow = \delta^\uparrow = \delta^\downarrow$

$\begin{array}{c|c}
1e & \delta_i = \pi/4 \\
2e & \delta_i = \pi/2 \\
\end{array}$

$G = G_0 \sum \sin^2(\delta_i)$

$G(2e) = 2G(1e)$

This type of arguments may be found for the SU(2) case in Glazman and Pustilnik cond-mat/0501007
Temperature dependence of conductance

Temperature 15.0 K

Conductance (e^2/h)

Barrier transparency grows

Vgate (Volts)

Ec, Δ ~ 100K

Shell spacing
Temperature dependence of conductance

Growth of the signal in the valleys due to the Kondo effect

Temperature
- 1.3 K
- 3.3 K
- 6.4 K
- 10.4 K
- 15.0 K

\( E_c, \Delta \sim 100 \text{K} \)
Shape of the 4-electron clusters at low T and high transparency

Single-electron features are washed away
Shape of the 4-electron clusters at low T and high transparency

Fabry-Perot (single-particle interference) does not work!
Shape of the 4-electron clusters at low T and high transparency

$G \sim \sin^2(\pi N/4)$ where $N$ varies linearly with $V_{\text{gate}}$
Electronic band structure, sample
Coulomb blockade
Kondo effect / Mixed Valence
- apparent Kondo behavior in the Mixed Valence regime
Phase shifts of scattered waves $\delta$

Low temperature singlet:

$\delta^{\uparrow} = \delta^{\downarrow} = \delta^{\uparrow} = \delta^{\downarrow}$

Friedel’s sum rule $\sum \delta_i = \pi N$

$N = CVg/e$ (open dot)

$$G = G_0 \sum \sin^2(\delta_i) = 4G_0 \sin^2(\pi CVg/4e)$$

Evidently, the argument works even for non-integer $N$

Characteristic energy scale $T_0 \sim 3 \text{ K} \ll E_0, \Delta, \Gamma$ (all $\sim 100 \text{ K}$)
Temperature dependence of conductance

Conductance (e^2/h)

Temperature
- 1.3 K
- 3.3 K
- 6.4 K
- 10.4 K
- 15.0 K

Ec, Δ ~ 100K
Spectroscopy at finite bias

Tunneling from the weakly coupled lead probes the density of states in the Kondo / Mixed Valence system.

\[ \Gamma_L \gg \Gamma_R \]

Width of the resonance: \( 10 \text{ K} \sim T_0 \)

Tunneling density of states

\( \text{E}_F \)
Spectroscopy at finite bias

Tunneling from the weakly coupled lead probes the density of states in the Kondo / Mixed Valence system.

\[ \Gamma_L \gg \Gamma_R \]
Electronic band structure, sample
Coulomb blockade
Kondo effect / Mixed Valence

Numerical Renormalization Group calculations

Leads and island formed within the same nanotube. 4 degenerate levels \((↑, ↓, ↑, ↓)\) in the dot are coupled one-to-one to 4 modes \((↑, ↓, ↑, ↓)\) in the leads. The modes are not mixed by tunneling; \(t\) amplitude does not depend on \(α\) or \(σ\). Hamiltonian has SU(4) symmetry.
NRG calculations of the SU(4) tunneling Hamiltonian

\[ \Gamma = 0.5 \text{ meV} \]

\[ \Gamma = 1 \text{ meV} \]

\[ \Gamma = 2 \text{ meV} \]

\[ T = 15, 8.5, 5, 3, 1.7 \text{ K} \]

(color sequence reversed)

F. Anders, M. Galpin, D. Logan, and G.F.
PRL (2008)
Zero-bias conductance – numerical results

\[ \frac{4e^2}{h} \sin^2\left(\frac{\pi N}{4}\right) \]

\( N \)

\( \Gamma = 0.5 \text{ meV} \)
Zero-bias conductance – numerical results

\[ \sigma = \frac{4e^2}{h} \sin^2\left(\frac{\pi N}{4}\right) \]

\[ \Gamma = 1 \text{ meV} \]

Different curves represent data taken at various temperatures:
- 15 K
- 8.5 K
- 5 K
- 3 K
- 1.7 K
Zero-bias conductance – numerical results

\[ \sigma = \frac{4e^2}{h} \sin^2 \left( \frac{\pi N}{4} \right) \]

\[ \Gamma = 2 \text{ meV} \]

\[ N = \text{Gate Voltage} \]

\[ \sigma (e^2/h) \]

\[ N_{\text{gate}} \]
Universal scaling curve $G(T/T_0)$ for 2 electrons

- $G(T)/G(0)$ vs $T/T_0$
- SU(2) and SU(4) curves
- Shells II, III, IV

2e SU(4) curve is more steep than in the 1e SU(2) case
Electronic band structure, sample
Coulomb blockade
Kondo effect / Mixed Valence
  -apparent Kondo behavior in the Mixed Valence regime
  -Kondo in magnetic field
Kondo in $B_\perp$: SU(4) Kondo suppressed

Non-degenerate ground state
Kondo suppressed
Kondo in $B_\perp$: SU(4) to orbital SU(2)

Degenerate ground state $\Rightarrow$ Orbital SU(2) Kondo
Nanotube in $B_{||}$

$E$  $B_{||}$

$V_{\text{gate}}$ (V)

Magnetic field (Tesla)

1e  2e  3e
2e singlet ground state => Kondo disappears

SU(2) spin Kondo still possible for 1e and 3e
Kondo in B∥

$T_K^{SU(2)} <$ spin splitting

$SU(4) \Rightarrow$ no Kondo

Singlet, non-degenerate

$T_K^{SU(2)} >$ spin splitting

$SU(4) \Rightarrow SU(2)$
Kondo features in 2 successive shells

\[ B_{||} = 3.5 \, \text{T} \]

- \( T_K^{SU(2)} > \text{spin splitting} \)
  - \( SU(4) \Rightarrow SU(2) \)
  - singlet, non-degenerate
  - \( SU(4) \Rightarrow \text{no Kondo} \)

- \( T_K^{SU(2)} < \text{spin splitting} \)
  - \( SU(4) \Rightarrow \text{no Kondo} \)
Kondo features in 2 successive shells

The difference between 1e and 3e cases is persistent in 2 successive shells

\[ B_{\parallel} = 3 \text{ T} \]
Spin-orbit

Without SO

Small spin gap
=> SU (2) Kondo

With SO

Large spin gap
=> no Kondo

Kuemmeth et al., Nature (2008)
Summary

- Orbital degeneracy in nanotube Quantum Dots
- Kondo effect / Mixed Valence – a **major** modification of conductance at low T.
- Why does $G \sim \sin^2(\pi N/4)$ work so well for noninteger $N$?
- What is the low energy scale?
- Lifting SU(4) symmetry by magnetic field:
  - $B \perp$ orbital SU(2) Kondo effect
  - $B \parallel$ spin-orbit in 1e and 3e valleys
Determination of $\Gamma$

\[ \frac{1}{(E-E_0)^2 - \Gamma^2} \]
Is the coupling of the two orbitals to the leads different?

The widths of the single-electron peaks at non-zero field for two orbitals in consecutive shells are about the same.

What is the cause of the e-h asymmetry?

Spin-orbit
Positions of the resonances

T = 1.3 K

0 1.2e²/h