For the α-particle $\langle x \rangle = \langle p \rangle = 0$, but there are fluctuations in the position and momentum. Roughly, the scale of the quantum fluctuations correspond to the classical range of $x$ and $p$. Therefore $\Delta x \approx$ diameter of the nucleus, and $\Delta p \approx p_{\text{max}} = \sqrt{2m_0E}$ (ignoring relativistic effects). From Heisenberg's uncertainty principle, $\Delta x \Delta p \approx \frac{\hbar}{2}$.

So that

$$\Delta x \approx \frac{\hbar}{2\Delta p} = \frac{\hbar}{2\sqrt{2m_0E}}.$$ 

From the chart $m_p c^2 = m_n c^2 = 940 \text{ MeV}$

$\Rightarrow m_0 = \frac{4 (940 \text{ MeV})}{c^2}$

$\hbar = 6.6 \times 10^{-27} \text{ MeV} \cdot \text{s}$

$\Delta x = \frac{6.6 \times 10^{-27} \text{ MeV} \cdot \text{s} \cdot 3 \times 10^8 \text{ m/s}}{2\sqrt{2 \cdot 4 \cdot 940 \text{ MeV} \cdot 6 \text{ MeV}}} \approx 0.5 \times 10^{-15} \text{ m} = 0.5 \text{ fm},$

which is in the right ballpark—about 1 fm.
c) The potential energy function is the same — its only the origin has been shifted.

What was a sine function in the old coordinate system is now a cosine function in the new system:

\[ \psi_n(x) = \sqrt{\frac{2}{L}} \cos \left( \frac{n \pi x}{L} \right) \]

a) The particle with the longer de Broglie wavelength will have the greatest chance of making it through the barrier.
The de Broglie wavelength is given by
\[ \lambda = \frac{h}{P} = \frac{h}{\sqrt{2mE}} \] for a non-relativistic particle.

Since \( E \) is the same for both the electron and the proton, \( \lambda_{\text{electron}} \neq \lambda_{\text{proton}} \) since \( m_e < m_p \). Therefore, electrons will have the highest probability for making it through the barrier.

- Penetration depth \( \delta = \frac{h}{\sqrt{2m(V_0 - E)}} \)

\[ V_0 - E = 10\,\text{eV} - 6\,\text{eV} = 4\,\text{eV} \Rightarrow \delta \approx \lambda \sin\alpha \]

From the table, \( m_e \approx 0.5 \,\text{MeV}/c^2 \), \( m_p \approx 940 \,\text{MeV}/c^2 \), \( k = 6.6 \times 10^{-16} \,\text{eV} \), \( c = 3 \times 10^8 \,\text{m/s} \)

\[ \lambda = \frac{6.6 \times 10^{-16} \,\text{eV} \cdot \text{s} \cdot 3 \times 10^8 \,\text{m/s}}{2\pi \sqrt{2(0.5 \times 10^6 \,\text{eV})(5\,\text{eV})}} = 1.14 \times 10^{-11} \,\text{m} = 0.14\,\text{\AA} \]

- Proton

\[ \lambda = \frac{6.6 \times 10^{-16} \,\text{eV} \cdot \text{s} \cdot 3 \times 10^8 \,\text{m/s}}{2\pi \sqrt{2(940 \times 10^6 \,\text{eV})(5\,\text{eV})}} = 3.3 \times 10^{-13} \,\text{m} = 0.0033\,\text{\AA} \]
Separation of Variables. The separation constants are related to conserved quantities, such as energy or angular momentum, and often take on discrete values (the quantum numbers).

A heterostructure laser is made from several thin layers of semiconductor materials, with differing bandgaps. The thickness of the layers can range from a few Å to a few μm, and often consist of GaAs and AlGaAs. Heterostructure lasers are used in fiber-optic communication systems, optical data storage, etc. Alferov & Kroemer won the 2000 Nobel Prize for their invention of the heterostructure laser.

From Schrödinger's time-independent equation,

\[ -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi = E\psi \]

\[ \frac{\hbar^2}{2mL^2} \]

\[ \frac{d\psi}{dx} = \frac{d}{dx} \left( Ae^{-x^2/2L^2} \right) = -\frac{2x}{2L^2} Ae^{-x^2/2L^2} = -\frac{xA}{L^2} e^{-x^2/2L^2} \]

\[ \frac{d^2\psi}{dx^2} = \frac{x^2A}{L^4} e^{-x^2/2L^2} - \frac{A}{L^2} e^{-x^2/2L^2} \]

Sub into Schrödinger's equation to find

\[ -\frac{\hbar^2}{2mL^4} \frac{x^2A}{L^4} e^{-x^2/2L^2} + \frac{\hbar^2}{2mL^2} e^{-x^2/2L^2} + V(x)Ae^{-x^2/2L^2} = \frac{\hbar^2}{2mL^2} e^{-x^2/2L^2} \]

\[ V(x) = \frac{x^2\hbar^2}{2mL^4} \]
The harmonic oscillator potential corresponding to Hooke's law force (spring):

\[ F = -\frac{dV}{dx} = -\frac{x}{mL^2} \]