Physics 143 - Second Quiz
November 6, 2000

I will abide by the Duke Honor Code     Name ________________________________

This is a closed book exam, with calculators and one crib sheet allowed. Only work shown in the blue book will be graded. However, please hand in these sheets with the blue book. TOTAL POINTS: 30.

1. (3 points) In a simple model for $\alpha$-particle decay of a radioactive nucleus, we picture the $\alpha$-particle rattling around the nucleus until it eventually tunnels through the potential barrier arising from the force that binds it to the nucleus. Estimate the size of a nucleus using only the Heisenberg uncertainty principle and the fact that the typical energy of an emitted $\alpha$-particle is 6 MeV. (You can ignore relativistic effects for simplicity.) How does your answer compare to what you expect for the size of a nucleus? [Hint: see attached constants and conversions.]

2. (6 points) A particle of mass $m$ is confined within a one-dimensional infinite square well potential given by

\[
V(x) = \begin{cases} 
\infty & x < 0 \\
0 & 0 \leq x \leq L \\
\infty & x > L 
\end{cases}
\]

The normalized spatial part of the wavefunction solutions for Schrödinger’s equation and this potential are given by

\[
\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}.
\]

(a) Make a sketch of the potential and the first few eigenfunctions.

(b) Consider the following potential energy function

\[
V(x) = \begin{cases} 
\infty & x < -L/2 \\
0 & -L/2 \leq x \leq L/2 \\
\infty & x > L/2 
\end{cases}
\]

Sketch the new potential energy function.

(c) What are the eigenfunctions corresponding to this potential? (No derivation is required if you can justify your answer).

3. (6 points) Electron of energy 5 eV and moving in a region of space where $V = 0$ are incident on a square barrier of height 10 eV and thickness 10 Å. After the barrier, the potential energy returns to zero. In a separate experiment, protons of energy 5 eV are incident on the same barrier.

(a) Which particle will most likely tunnel through the barrier, or are the just as likely to tunnel through? Explain.

(b) Estimate the penetration depth of the electrons and protons into the barrier. [Hint: see attached constants and conversions.]
4. (3 points) What is the mathematical technique we use to transform a partial differential equation into a set of coupled ordinary differential equations? Explain the physical significance of the constants introduced during this procedure.

5. (3 points) Briefly describe what is meant by a heterostructure laser.

6. (3 points) What is the relation between the probability for reflection $|R|^2$ and the probability for transmission $|T|^2$ of a particle through a thin barrier?

7. (6 points) In a region of space, the spatial part of the wavefunction describing a particle of mass $m$ is given by

$$\psi(x) = Ae^{-x^2/2L^2}$$

and its energy is

$$E = \frac{\hbar^2}{2mL^2}.$$

(a) Find the potential energy function $V(x)$ corresponding to the force acting on the particle.

(b) Sketch $V(x)$, making sure to label important aspects of your sketch.

(c) What is the classical potential that has this dependence? What force does it correspond to?