Often thermodynamic functions involve Legendre transformations and Maxwell relations.

We have encountered a collection of thermodynamic "potentials" including $F$, $G$, $H$, and $I$. Each of these contains the same information as $U(T,V,N)$, when expressed in terms of their respective natural variables.

The connections among potentials involve Legendre transformations. The basic idea of an L.T. is to represent the same information given by a function $y(x)$ in terms of a different variable, namely the slope $s = \frac{dy}{dx}$.

The L.T. takes the original function $y(x)$, and computes the intercept $f(s)$. This process is reversible (for well-behaved $y(x)$).

$$f = y - sx = f(s)$$
Now let's look at the various terms that have appeared.

\[ F = U - \pi \tau = F(\tau, V, N) \]

\[ dF = -\tau d\pi + \beta dV + \mu dN \]

This yields the Maxwell relation

\[ \left( \frac{\partial F}{\partial V} \right)_T = \left( \frac{\partial \mu}{\partial T} \right)_N \]

We have looked briefly at a further transformed function

\[ G = F - (\pi V) = F + \beta V = U - \pi \tau + \beta V \]

\[ G(\tau, V, N) \]

This is the Gibbs free energy and

\[ dG = -\tau d\pi + \beta dV + \mu dN \]

Maxwell relation:

\[ \left( \frac{\partial V}{\partial \beta} \right)_\pi = -\left( \frac{\partial \mu}{\partial \tau} \right)_T \]
We have also considered the enthalpy:

\[ H = U - (pV) = U + pV = \mathcal{H}(\xi, p, N) \]

\[ dH = \tau d\xi + Vdp + \mu dN \]

Maxwell relation \( \left( \frac{\partial \tau}{\partial p} \right)_\xi = \left( \frac{\partial V}{\partial \mu} \right)_p \)

Finally we considered the Grand Canonical Potential:

\[ \Xi = U - \beta S - \mu N = \mathcal{\Xi}(\beta, \mu, N) \]

\[ d\Xi = - \beta dS - \mu dN - N dp + V dp = N dp \]

Maxwell relation \( \left( \frac{\partial \beta}{\partial \mu} \right)_N = \left( \frac{\partial V}{\partial S} \right)_{\mu, N} \)