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Making a Rough Place "Plane": Why Heaping of Vertically Shaken Sand Must Stop at Low Pressure

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Abstract The heaping of a granular material subject to vertical vibration vanishes abruptly as the pressure of the surrounding gas, P, is lowered below a critial value ~ 10 Torr, depending on particle diameter etc. We show that the vanishing of the heap is consistent with two different effects. One of these is the onset of a Knudsen regime where the mean free path of a gas molecule becomes comparable to or larger than the typical distance to a grain. The usual Darcian gas flow models fail in this regime, and a Knudsen replacement predicts a vanishing of gas effects as $P \to 0$. The other is that at low enough pressures, there is not enough gas to sustain flow under the usual linearized flow scenario. A detailed description of this regime is beyond the present analysis.

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1 Introduction

Recently, there has been substantial interest in granular flows[1]. Although many such flows depend only on the properties of the grains and their interactions with the boundaries, there are a number of cases where the surrounding gas can play an important role. Examples of the latter include vertically shaken materials and gravity driven flows such as those in hourglasses or pipes[2–7]. Effects from the surrounding air or other gas become important when the grain size is small enough, typically for grain diameters, $d \lesssim 1$ mm. In this work, we consider, in the context of shaker systems, what happens to pressure effects as the gas pressure, P, is reduced toward zero. The issues raised here should also be relevant broadly to systems where gas is important and where the pressure is low[8].

For the shaker system, the presence of gas leads to the formation of a heap, as first discussed by Faraday[2]. If all the gas is removed, the heap goes away[4,5]. If the pressure, P, is decreased monotonically towards zero, the heap height, H, grows first to a slight maximum until a characteristic pressure, below which the heap height falls off abruptly. In spite of much study, there has been no discussion of the physics that is responsible for these features.

Here, we focus on two possible reasons for the rapid fall-off of H with decreasing P. The first of these is the onset of a Knudsen regime which occurs when the typical distance between a gas molecule and a grain becomes comparable to a gas molecule mean free path (mfp),

$$\ell \simeq 1/n\sigma, \tag{1}$$

where n is the number density of molecules, $n = P/RT$, and σ is the cross section for intermolecular collisions. We estimate that the onset of this regime coincides with the falloff of heaping in recent experiments[5]. In order to describe this regime, we use a Knudsen-based replacement for the usual linearized Darcian flow equations. This Knudsen regime model predicts the vanishing of pressure effects, unlike the Darcian model. However, simple linearization of the Darcian flow equations also fails in roughly the same pressure regime, and this is the second issue that we consider here. This means that a description of the transition between the Darican and Knudsen regimes is complicated and hence beyond the scope of this work. However, it is possible to estimate the forcing available from flow in the linear Darcian regime and the Knudsen regime, as shown here.

The system which we consider is schematicized in Fig. 1. A box of granular material of mean height, h undergoes a vertical displacement $z = A\cos(\omega t)$. (In many of our experiments, the rectangular box was replaced by an annular containaer in order to remove the frictional forcing from one pair of sidewalls.) A heap forms for moderate dimensionless accelerations,

$$\Gamma = A\omega^2/g, \tag{2}$$

with $\Gamma \gtrsim 1$. We characterize this heap by either the distance L above the base of the container (as measured when the heap is in contact with the base) or the angle Θ of the heap relative to horizontal. The layer also goes into free flight during part of each shaker period. This creates a gap between the base of the shaker and the bottom of the granular material. During the cycle, gas flows in and out of the material with a characteristic speed that is at least nominally set by the peak velocity of the shaker: $v_{shaker} \simeq A\omega$. The heap is one manifestation of a complex flow which involves not only the effects of gas, but also friction with the sidewalls and dilational aspects[9, 10]. For instance, the complexity of the flow is reflected in the fact that for rough particles, there is an upflow of grains along the side of the container, whereas for smooth grains, there is a downflow[10]. The effect of overall gas flow has also been studied by Akiyama et al.[11] who found that changes in the shaker geometry that affected the gas flow also affected the heaping process.

Fig. 2 shows the variation of heap height and angle of inclination, Θ, with P. Part a of this figure shows Θ over a large range of P, and part b shows L for just the small-P region. We use two measures of the heap height, H, and L, which are simply related by $L = h + H/2$. Here, h is the mean height of the layer, which can be read off the bottom part of Fig. 2, since as $P \to 0$, $L \to h$. L and Θ change relatively slowly with decreasing P until

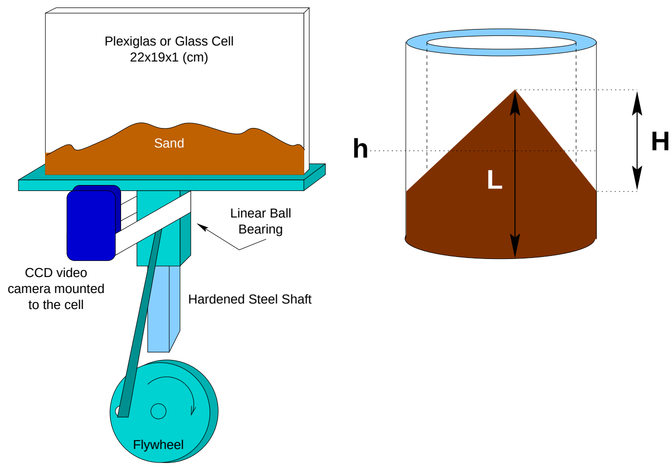

Fig. 1. Schematic of the experiment, showing various parameters. Note that $L = h + H/2$.

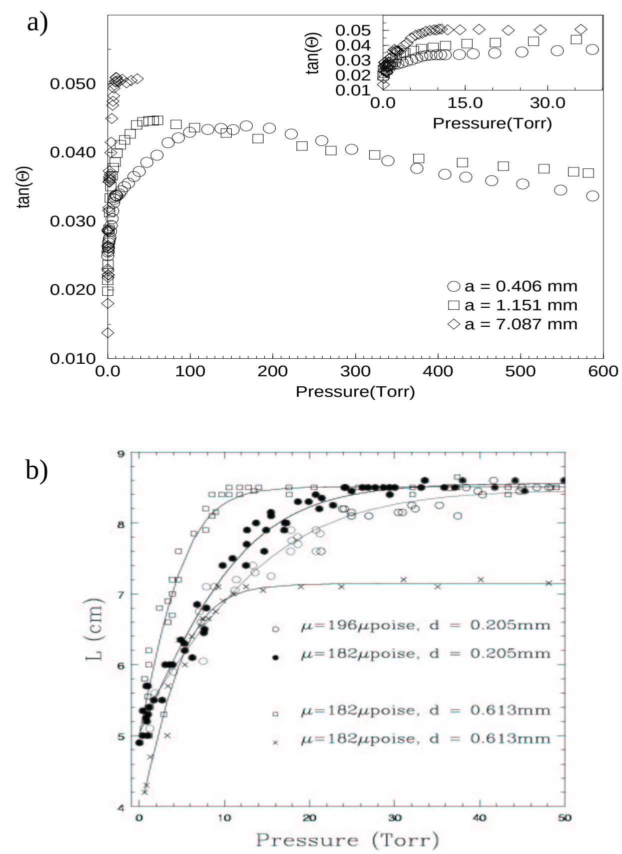

Fig. 2. a) Angle, Θ, of the granular heap relative to horizontal vs. Pressure, P after Pak et al. Different symbols indicate different shaking amplitudes, A. b) Heap-to-base height, L vs. P for a smaller range of P and for various grain diameters, d. Different symbols correspond to the indicated viscosities (196 and 182 $\mu poise$, respectively for helium and air) and d's. The data were obtained at $\Gamma = 1.3$ with $A = 5.34$ mm.

$\sim$ 10 Torr; below this pressure, they decrease rapidly[5, 10]. At very low P, $P \leq 10^{-4}$ Torr, there is only a small residual heap which apparently remains because of wall forcing. In Fig. 2b, we have fitted the data near the falloff region to the form $L = a \tanh(KP) + b$, so that K^{-1} is a typical pressure at which the heaping stops. Note that the data set in this figure which is lower than the others corresponds to a lower mean height, h, of the layer.

A conventional way to model gas flow through the granular material is to assume that the material is a porous medium (PM) characterized by Darcy's law[3]. Thus, there is a solid matrix of grains (which evolves in time) surrounded by a pore space occupied by gas. This kind of model captures several of the relevant features. However, at low P, mfp effects can become important, indicating a failure of this model. A simple calculation outlined below indicates that these effects become important just at the point where L starts to fall off in Fig. 2.

In order to provide additional insight into this system, we show in Fig. 3 particle tracking images where individual particles were followed over time, and in Fig. 4 information on the overall flow patterns. Fig. 3 shows in particular, that there is a strong flow of particles inward under the base of the heap. These particles are then pushed upwards when the layer of material collides with the base of the shaker, providing the nominal "motor" to drive the heaping process, as pointed out by Laroche et al.[4]. Fig. 4 shows how the system evolves in time by constructing lay-

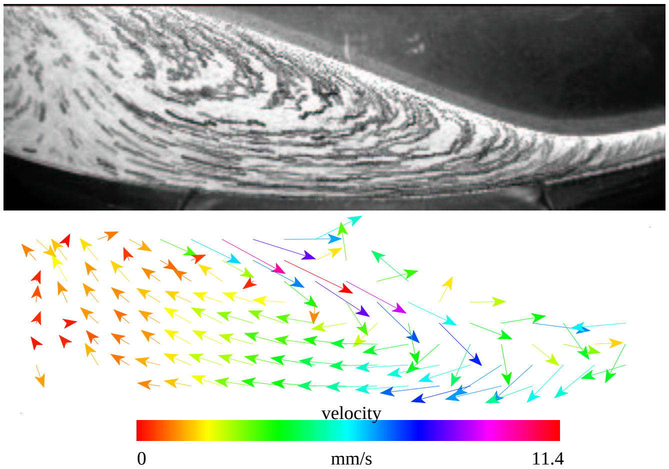

Fig. 3. Twelve second time-lapse image showing the motion of individual grains near the base of the container and the valley of the heap. Grains are pulled in rapidly under the base. Overlayed on the image is the velocity field color coded according to magnitude. These results are for atmospheric pressure.

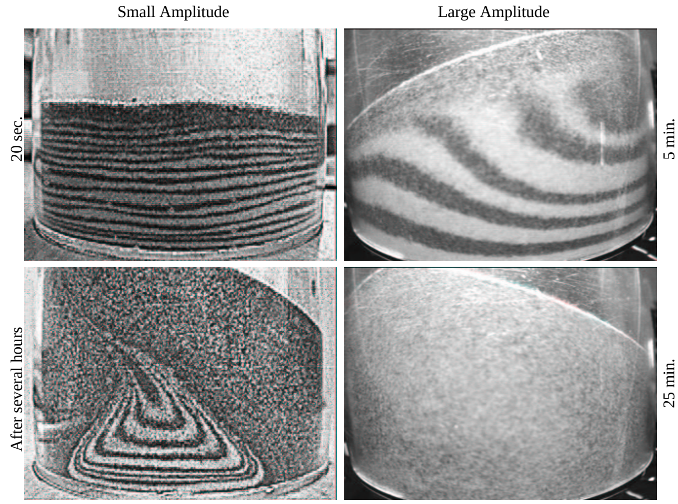

Fig. 4. Time-lapse images showing the flow of material for both large amplitude ($A = 0.66\,\mathrm{cm}$) and small amplitude ($A = 0.089\,\mathrm{cm}$). The left two images are the initial (top) and final (bottom) states for the smaller shaking amplitude, and the right two images are shortly after the initial (top) and final (bottom) state for the larger amplitude. The initial state in each case consisted of a series of light and dark material of the same size. The amount of material convected under the heap increases with amplitude, A. For the small amplitude, the central core region remains unmixed after $\sim 10^6$ shakes.

ers with initially flat profiles built from alternate dark and light colored layers. The figure contains information for relatively large and relatively small A, and shows also that the origin of the heaping occurs as material near the valley is advected under the heap. Note that a valley reaching to the container base typically forms in the annular container nearly opposite the peak, provided h is not too large. For a rectangular container, the corresponding location for the low part of the layer is typically at the extreme edges. At the valley, the particles move towards the center of the heap which can only be due to pressure gradients. When A is small, this effect is reduced, and the central portion of the material may remain unperturbed for very long times; in the small-A images, the central region is unmixed after more than 10^6 shakes.

Before turning to an analysis of the pressure gradients which may result from the flow, we consider when a Knudsen regime should begin, and we show that this corresponds reasonably well with the falloff of heaping in experiments as $P \to 0$. It is clear that as P becomes small, the mean free path of the gas will become large enough that collisions of molecules with grains will be at least as important as collisions with gas molecules. We frame this issue by asking when the typical distance, D_{open}, between a point in the open pore space and a grain is equal to ℓ. We calculate D_{open} for a dense packing by the following simplification. We note that for each solid volume,

$$V_s = (4/3)\pi(d/2)^3, \tag{3}$$

(where d is a grain diameter) there is a corresponding open pore volume of

$$V_o = [\phi/(1-\phi)]V_s, \tag{4}$$

where ϕ is the porosity or open volume fraction. We approximate V_o as a sphere of radius $R = (d/2)[\phi/(1-\phi)]^{1/3}$. By a simple calculation, the average distance between a point inside a sphere of radius R and its boundary is then

$$D_{open} = R/4 = (d/8)[\phi/(1-\phi)]^{1/3}. \tag{5}$$

Obviously, this calculation is simplistic, but for a dense packing, the typical distance between a pore space point and the wall is a relatively small factor of d due to geometric considerations, say:

$$D_{open} \simeq 0.1d. \tag{6}$$

We can then compare the onset of mfp effects with the fall-off of heaping using

$$\ell = (n\sigma)^{-1} = k_B T/P\sigma. \tag{7}$$

For Nitrogen gas at 10 Torr, and $d = 0.05$ cm, $\ell/0.1d \simeq 0.6$. More generally, the condition $\ell = 0.1d$ implies an onset for mfp effects at:

$$P_{mfp} = 10k_B T/\sigma d. \tag{8}$$

2

Pressure Effects in a Linearized Darcian Model

We turn next to an estimate of the pressure forces which may be present during the shake cycle for an ordinary viscous gas. (For a more extensive calculation, see Gutman[3].) An exact description of the flow is a formidable task well beyond the present work. In order to make progress, we frame a simpler problem, examining the properties of its solution and possible implications for granular heaping. Specfically, we consider a layer of granular material of height h, which we model as a PM which is subject to a sudden change in gas pressure (or velocity) at one end (the bottom, $z = 0$). The typical velocity step is taken to be of the same size as what we would expect from the velocities set by the shaker, i.e. $v_{shaker} \sim A\omega$. We then estimate the typical force which a grain would experience if it were subject to such a step. It is interesting to compare this force to the weight of a grain, and to estimate how far such a force could carry a grain during the time that the granular material is in free flight above the shaker, if the force were applied horizontally. We carry out this analysis for Darcian flow in this section, and similar analysis for Knudsen flow in the following section. In the latter regime, ordinary viscous drag vanishes, and the Darcian model for porous media flow cannot apply. Regardless of the regime, a step change in the gas velocity leads to a damped flow of gas which typically decays on a time scale which is shorter than the time for one shaker period, T. Accordingly, we factor in the decay of the resulting pressure gradient as well.

For the Darcian picture[3,5], we approximate the gas flow through the PM by Darcy's law,

$$B(\rho/\phi)\partial v/\partial t = -[\partial P/\partial z + (\mu/\gamma)v] \tag{9}$$

and the continuity equation

$$\phi\partial\rho/\partial t = -\partial(\rho v)/\partial z, \tag{10}$$

where ρ and μ are the gas density and viscosity, v its velocity relative to the PM, and γ its permeability. We assume the Ergun relation for the last of these:

$$\gamma = d^2\phi^3/[150(1-\phi)^2]. \tag{11}$$

For a typical grain size, $d = 0.05\,\text{cm}$, and $\gamma = 3.0 \times 10^{-6}\,\text{cm}^2$. In Darcy's law, we have included an acceleration term, $B \neq 0$, in order to avoid violating causality, but more extensive analysis shows that the normal version (with the left side replaced by zero) is actually adequate here. For typical gas velocities $\sim 25\,\text{cm/s}$, and at atmospheric pressure, the pore-scale Reynolds number is $Re = v_{shaker} d/\nu \simeq 10$ (ν is the gas kinematic viscosity). This is above the regime where Darcy's law is strictly reliable, but the corrections are not likely to be significant at the level of the present approximations. By the time that P falls to $\sim 0.1\,\text{Atm} = 76\,\text{Torr}$, we expect $Re \simeq 1$, for which the Darcian flow model should be reliable. Moreover, we assume[3] that small variations in ρ are related to those in P by the isothermal compressibility because the grains effectively provide a heat bath:

$$\delta\rho = \rho\kappa_T\delta P. \tag{12}$$

Appropriate manipulation yields a diffusion equation for small amplitude variations in any of v, P, or ρ. For example:

$$\partial\delta P/\partial t = D_D\partial^2\delta P/\partial z^2, \tag{13}$$

where $D_D = \gamma/(\phi\kappa_T\mu) \propto P$. For a set of standard conditions ($P = 1\,\text{Atm}$, $d = 0.05\,\text{cm}$, $\phi = 0.4$, $\mu = 1.82 \times 10^{-4}\,\text{poise}$, $A = 0.5\,\text{cm}$, $h = 10\,\text{cm}$, $\Gamma = 1.3$), $D_D = 4.1 \times 10^4 cm^2/s$.

We look for normal mode solutions to these equations where the space-time varying parts of P, v etc. have the form

$$(\delta P, v) = (P_o, v_o)\exp(ikz - t/\tau), \tag{14}$$

with amplitudes related by

$$P_o = iv_o\mu/(k\gamma). \tag{15}$$

To make the problem more precise, we will assume approximately realistic boundary conditions, namely $v = 0$, i.e. a no-gas-flux boundary condition at the bottom of the layer ($z = 0$), and a fixed-P boundary condition at the top ($z = h$). That is, $\delta P = 0$ at $z = h$. Implementing the boundary conditions we obtain solutions for the pressure part of each mode:

$$P_n = P_{on}\cos(k_n z)\exp(-t/\tau_n), \tag{16}$$

where

$$k_n = \pi(2n - 1)/2h, \tag{17}$$

is a positive integer, and

$$\tau_n = 1/D_D k_n^2. \tag{18}$$

Of these, the slowest one, $n = 1$ dominates. We henceforth consider only this mode, with relaxation time

$$\tau = (2h/\pi)^2/D_D, \tag{19}$$

and drop any subscript.

We would like to calculate the force on a grain from the gas flow. In principle, this requires the stress tensor at the grain surface, something which we do not know. However, we estimate the force on a grain by

$$F_{gas} = \int P d\mathbf{a} = (\pi/6)d^3 P_o k. \tag{20}$$

It is also important to recognize that D_D is relatively large, so that τ may be short compared to the shaker period, $T = 2\pi/\omega$. For example, in a typical experiment at atmospheric pressure, $T \simeq 0.1\,\text{s}$, and $\tau \simeq 10^{-3}\,\text{s}$. In that event, a more meaningful estimate of F_{gas} involves an average over the exponential time decay, i.e. the mean force on a grain should be reduced by the integral

$$I(T/\tau) = \frac{1}{T}\int_0^T \exp(-\frac{t}{\tau})dt = \frac{\tau}{T}(1-\exp(-\frac{T}{\tau})). \quad (21)$$

In principle, only the free flight time of the material should be used, so that T should be reduced as $\Gamma \to \Gamma_c$. However, this level of detail is beyond the current analysis. I varies between 1 (τ/T large) and τ/T, ($\tau/T << 1$). At atmospheric pressure, the latter is likely to apply, but at small enough P, $I \to 1$.

In order to estimate the effect of F_{gas} according to this model, we calculate the ratio, R_D, of F_{gas} to the force of gravity, $mg = (4\pi/3)(d/2)^3\rho_g$, determined by the bulk density, ρ_g, of the grain material:

$$R_D = \frac{F_{gas}}{F_{gravity}} = \frac{P_o k I}{\rho_g g} \simeq 1.8\times 10^4 (h\mu/d^2)^2 \Gamma/\rho_g P. \quad (22)$$

Here the rightmost expression applies in the small-τ (typically large-P) limit. This expression provides an order of magnitude estimate which has some qualitatively correct features, but also some features which require additional discussion. It correctly indicates that pressure effects should depend strongly on particle size, and that they should actually become stronger with decreasing P, which is in fact the case experimentally for pressures above K^{-1}. However, it does not predict the cessation of heaping at very low P, nor does it indicate why the amplitude, A, should be important. Also, $R_D \simeq 0.006$ at $P = 1\,\text{Atm}$ (standard conditions), but is nearly two orders of magnitude larger at P_{mfp}, whereas the observed variation of L or Θ with P is much weaker.

An explanation for amplitude effects comes by examining the motion of grains which are carried by gas pressure effects under the heap in the vicinity of the valley. In general, we expect that a horizontal force $\sim F_{gas}$ acts on the grains near the base during the time that the heap is in free flight, i.e. for a time $T\Delta$, where Δ is the fraction of a period which the layer spends off the base. For example, $\Delta = 0.46$ for $\Gamma = 1.3$, a typical Γ for these experiments. This estimate for Δ comes by assuming that the distance between the base of the shaker and the layer can be described by the trajectory of an inelastic ball driven by a sinusoidally oscillating platform[13]. Although this seems on the surface to be an extreme approximation, in practice, we have found that it is a reasonable approximation of reality[12]. Hence, we would expect that during each shake cycle, there would be a lateral displacement, $x = (F_{gas}/m)(T\Delta)^2/2$, towards the center of the heap, given in units of d by

$$x/d = 2\pi^2\Delta^2 R_D (A/d)\Gamma^{-1}. \quad (23)$$

For the standard conditions, $x/d \simeq 0.21$, not a large value. Perhaps more importantly, the mean speed associated with this displacement is $x/T \propto A^{1/2}$. Since the distance which the bottom of the heap reaches above the base also increases with A, this expression suggest at least heuristically, why the heaping effect increases strongly with amplitude.

3

Failure of the Darcian Picture at Low Pressures

The linearized Darcian picture fails as $P \to 0$ for two reasons: first, viscous flow no longer applies as $\ell \to d$, and second, the pressure amplitude needed to obtain a velocity of $\sim A\omega$ is comparable or larger than the ambient pressure. We consider first the Knudsen regime. In this case, diffusion of gas through the material still occurs with a mfp set by the grain spacing and a collision time

$$\tau_c \sim d/v_T, \tag{24}$$

where

$$v_T = (8k_BT/\pi m)^{1/2} \tag{25}$$

is the mean thermal velocity of the gas molecules of mass m. That is, we think of a molecule as executing a random walk through the material with a step size $\Delta x \simeq d$ and a step time τ_c. The result is diffusion with a diffusivity

$$D_K \simeq v_T d. \tag{26}$$

This limit is particularly relevant in industrial applications where there are very fine particles[8]. The interesting distinction between the fine powder case and the present situation is that here, the forces from the gas flow are weak relative to gravitational forces (i.e. as $P \to 0$). We can estimate the prefactor in this result by making the Darcian and Knudsen diffusivities identical, $D_D = D_K$, when $\ell = 0.1d$. The result is

$$D_K = (\pi/40)(\phi/(1-\phi))^2 v_T d. \tag{27}$$

In the presence of a density gradient (i.e. pressure gradient), D_K replaces D_D above. We expect a mass flux

$$j = -D_K \nabla\rho, \tag{28}$$

which defines a velocity field via

$$j = \rho\phi v. \tag{29}$$

Here, we assume that the mean free path of a molecule is small compared to the dimensions of the container, so that a continuum approximation is still valid. The continuity equation, $\partial\rho/\partial t + \nabla\cdot j = 0$, implies a diffusion equation for P, v, ρ, etc. where now the diffusivity is D_K. As before, we use $\delta\rho = \rho\kappa_T\delta P$. We can then repeat the previous analysis to obtain the typical force on a grain in this regime:

$$R_K = F_K/F_{gravity} = v_o I(T/\tau)/\phi\rho_g D_K\kappa_T \propto P. \tag{30}$$

A key point is that D_K is independent of P, unlike D_D in the viscous case[8]. Now, the effects of the pressure vanish with decreasing P, as needed. Using the standard conditions except $P = 10\,\mathrm{Torr}$, with $T = 293\,\mathrm{K}$, yields $D_K = 580 cm^2/s$; $R_K = 0.17$. In the same vein as the Darcian calculation we estimate a typical horizontal displacement per shake of

$$x/d = 2\pi^2\Delta^2 R_K(A/d)\Gamma^{-1}. \tag{31}$$

The second issue that arises as $P \to 0$ is that the simple linearlized Darcian picture in which we assume that the gas velocity is $\sim A\omega$ breaks down in roughly the same regime of pressures as P_{mfp}. That is, the assumed pressure amplitude, $P_o = iv_o\mu/(\kappa\gamma)$, becomes comparable to ambient pressure at about the same time that $\ell \simeq 0.1d$. Specifically, for the standard conditions used above, the pressure amplitude is $P_o = 7.1\,\mathrm{Torr}$. The assumption of Eq. 12 that there is a simple linear relation between pressure and density fluctuations, with a constant proportionality coefficient, must be abandoned to describe pressures of this size. The Knudsen picture must be used at low enough pressures, but in the crossover between Knudsen and Darcian regimes, a full nonlinear treatment of both gas and particle dynamics is necessary to resolve which effect plays the most important role.

For typical experiments, the points at which $P_o \simeq P$, and at which $\ell \simeq 0.1d$ are close together. However, it is possible to separate these two pressures and hence to examine the two effects separately. Specifically,

$$P_o/P_{mfp} = (40/\sqrt{2}\pi)(v_o/v_B)(1-\phi)^2/dk\phi^3, \tag{32}$$

where we have used $v_B = (8k_BT/\pi m)^{1/2}$, and the Ergun relation for γ. For the standard conditions, $P_o/P_{mfp} = 0.72$. By adjusting the temperature, v_{shaker}, the particle diameter and the layer height, it should be possible to observe the two effects separately. We anticipate this type of study in the future.

4

Conclusions

To conclude, we have shown that the decrease of granular heaping as $P \to 0$ is connected to two effects. One of these is the onset of a Knudsen regime where the mfp is comparable to or larger than the distance between a gas molecule and a grain. This applies in the low pressure limit. We have then considered a replacement model for gas flow in this regime and shown that it has the correct properties to account for the end of heaping in the low-P limit. In roughly the same pressure regime of existing experiments, a linear description of Darcian flow, as used in past calculations[3, 5], breaks down. Necessarily, we have used simplified analysis, and there remains the considerable challange of describing the detailed coupled nonlinear granular-gas flow. Such an analysis would include the fact that as the amplitude (or Γ) increases, there is more dilation of the upper layer, and a greater open volume under the bottom of the heap. The first of these leads to increased surface grain flow, and second to stronger gas flow. Also associated with an increased A is an increase in the effect of shearing at the sidewall. An interesting possibility might be the use of molecular dynamics for both particles and for grains to better address the Knudsen regime.

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