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Memory in two-dimensional heap experiments

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The measurement of force distributions in sandpiles provides a useful way to test concepts and models of the way forces propagate within noncohesive granular materials. Recent theory [J.-P. Bouchaud, M.E. Cates, and P. Claudin, J. Phys. I **5**, 639 (1995); M. E. Cates, J. P. Wittmer, J.-P. Bouchaud, and P. Claudin, Phil. Trans. Roy. Soc. **356**, 2535 (1998)] by Bouchaud *et al*. implies that the internal structure of a heap (and therefore the force pathway) is a strong function of the construction history. In general, it is difficult to obtain information that could test this idea from three-dimensional granular experiments except at boundaries. However, two-dimensional systems, such as those used here, can yield information on forces and particle arrangements in the interior of a sample. We obtain position and force information through the use of photoelastic particles. These experiments show that the history of the heap formation has a dramatic effect on the arrangement of particles (texture) and a weaker but clear effect on the forces within the sample. Specifically, heaps prepared by pouring from a point source show strong anisotropy in the contact angle distribution. Depending on additional details, they show a stress dip near the center. Heaps formed from a broad source show relatively little contact angle anisotropy and no indication of a stress dip.

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Granular materials are of great interest for their technical relevance and for the host of modeling challenges that they present [1]. Transporting and processing agricultural grains, coal, and pharmaceutical powders represent only a few applications. Avalanches and mudslides are important natural phenomena that involve granular materials. Our understanding of granular flow, mixing, and even static behavior is still an open subject. In this paper, we focus on the last of these.

Static or slowly evolving granular systems are dominated by stress chains, long filamentary structures corresponding to the paths along which the majority of the force is carried [2]. In the photoelastic image of Fig. 1(b), the bright disks are part of the force chains in a two-dimensional (2D) realization of a sandpile. The formation of these chains plays an important role in the final static state of a granular system. One reason for this history dependence is that the solid-on-solid (SoS) friction between individual particles is typically indeterminate. That is, under static conditions, the tangential frictional force at a contact, F_T, can be anywhere in the range $|F_T| \leqslant \mu |F_N|$, where μ is the ordinary SoS friction coefficient and F_N is the normal contact force. For collections of stiff frictional grains, it is not always possible to determine the forces at the contacts solely from the positions of the grains [3].

Recently, several authors have proposed and/or discussed new continuum models for stresses within sand piles, including the oriented stress linearity (OSL) model [4], which is predicated on a picture of how force chains are frozen into a granular system during its formation, thus creating inherent textures. This model was created, in particular, to better understand reported stress minima [5,6] under the center of some sand piles. There are other models for granular statics [7–9], some of which also can predict a stress dip, including ''incipient failure everywhere'' (IFE), which assumes that the heap is everywhere at the point of Coulomb failure and elastoplastic models. The stress dip has also been studied via discrete element models [10,11] (DEM) modeling techniques, including molecular dynamics (MD) and contact dynamics [12] (CD). In MD models, a stress dip was found [10,11] depending on construction history, or in one case [11], if strong segregation occurred for piles formed from a bidisperse collection of particles. CD calculations [12] predict anisotropy in the contact angles between neighboring particles that reflects the construction of the heap.

The existence of force minima under the base of real sandpiles [5,6,8,13,14] has been equally debated [8,15]. Considerable care must be taken with experiments because per-

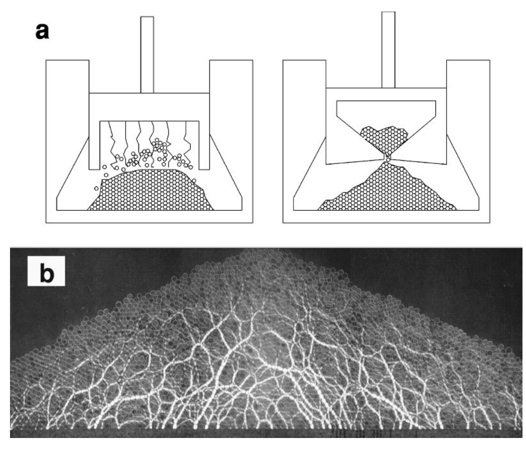

FIG. 1. Lower section: Setup of a two-dimensional pile (height ∼30 cm and base length ∼130 cm) of photoelastic disks created by a localized-source procedure. The pile is viewed between crossed polarizers, allowing one to see the underlying force structure. Bright regions correspond to the force chains. Upper section: Deposition procedures for triangular piles. Left picture shows raining technique, right shows the localized procedure. See text for more details.

turbations, such as small deflections of the base surface, may significantly affect the measured profile. Recent measurements [14] on 3D conical piles that had minimal deflections of the base, showed a clear dip when the material was poured on the pile from a localized source, and no dip when the pile was created by pouring from an extended source. Additional experiments [14] on long 3D heaps (mountain chain shape) showed similar, but weaker force sensitivity, to how the heap was formed.

Here, we present measurements of contact angle and force distributions for 2D heaps prepared by pouring particles from (i) a fixed height pointlike source; (ii) a slowly moving point source; and (iii) an extended source. There is a dramatically different arrangement of contact angles for the point versus extended sources, even though the heap angles for the two techniques are essentially indistinguishable. There is also a clear difference in the force profiles for the various preparation methods. For the fixed height localized source, the distribution shows a clear minimum in the force at the heap center. For the other two methods, the force distribution shows a broad maximum at the center, and possibly other structure on a finer scale.

The experiments were carried out with disks made from a photoelastic (birefringent under stress) material [16]. The sample was a mixture of two disk sizes, one with diameter $=0.9$ cm (~ 500 disks) and the other with diameter $=0.7$ cm (~ 2500 disks). The disks were confined between two Plexiglas sheets, spaced slightly wider than the thickness (0.6 cm) of the disks (the flat sides of the disks were parallel to the Plexiglas). The surfaces of the parallel sheets were lubricated with a fine powder and typically oriented no more than $3°$ from vertical to minimize friction between the sheets and the disks. The heap size was ~ 130 cm at the base, and ~ 30 cm high. We built heaps by the three different pouring techniques, two local and one extended, as sketched in Fig. 1(a). For the local source, the disks were held in the hollow Plexiglas insert between the two confining Plexiglas sheets. The insert had an opening ~ 7 grain diameters wide, small enough to be pointlike, but wide enough that flow-stopping arches rarely formed. In the fixed height version of the local source, the filled insert was placed at a fixed height, 57 cm, above the base. A stopper was then removed from the insert opening, and the disks flowed out. In the slowly moving version of the point source, we gradually raised the insert, creating a steady slow flow of disks onto the peak of the emerging pile. The extended source also consisted of an insert, but the central portion contained strips of either cardboard or Plexiglas. When the insert was lifted, it produced a steady rain of particles. As the heap formed, some of the particles then avalanched off, and the final heap profile was not perceptibly different from those formed by the point-source method. In all cases, we imaged the final state of the system with video (640×480 pixels). In order to obtain high resolution, we obtained two sets of three overlapping images that covered the central region of the heap, ~ 110 grain diameters wide. One set of images was obtained with the system placed in a circular polarimeter [17] and one without the polarimeter. In obtaining the various sets of images, the particles were left undisturbed for a given realization. The images with polarimeter yielded force information using an application of photoelasticity described elsewhere [18] and the other yielded the disk positions. We analyzed the images without the polarimeter to obtain the particle centers, and hence, the contact angles. A given production of a heap showed large variations in the spatial structure of the contacts and stress chain network. In order to obtain a reasonable statistical average, we carried out 50 realizations of each heap preparation method.

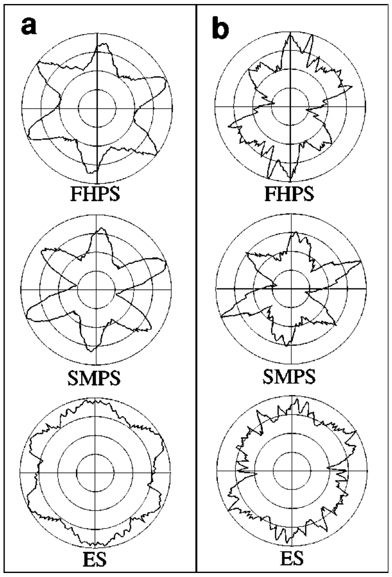

FIG. 2. (a) Comparison of mean-nearest-neighbor-angle distributions, $\rho(\theta)$, of all particles for three different pouring techniques: fixed-height point source (FHPS), slowly-moving point source (SMPS), and extended source (ES). The distributions are only for the left side of the sample typified by Fig. 1(b). The $\rho(\theta)$ are given in radial plots and scaled in each plot such that the maximum value is 1 (the outmost circle). (b) Comparison of mean-nearest-neighbor-angle distributions for the particles on the force chains for three different pouring techniques. The distributions are for the left side of the sample too.

In order to provide a simple measure of the contact orientation, we evaluated the average angular distribution of contacts, $\rho(\theta)$, for the left and right sides (about the vertical center line) separately. The orientation of the particle contacts was dramatically different for the various preparation methods. We consider a contact to exist if the distance between the centers of two partciles is within 4% of R_1+R_2, where R_i are the radii of the two particles. Thus, some of the contacts may not be force-bearing. We only consider data for the left half of the heaps, since similar distributions result for the right side with an appropriate mirror reflection. In Fig. 2(a) we contrast the distributions for the fixed height point source (FHPS), slowly moving point source (SMPS), and the extended sources (ES). For both the localized-source procedures, there is strong anisotropy, and a clear preferred set of orientations. By contrast, for the extended source procedure, the contacts are much closer to having an isotropic orientation of contacts.

We have also analyzed the contacts for only those disks that lie on a stress chain, i.e., the distributions of neighbors

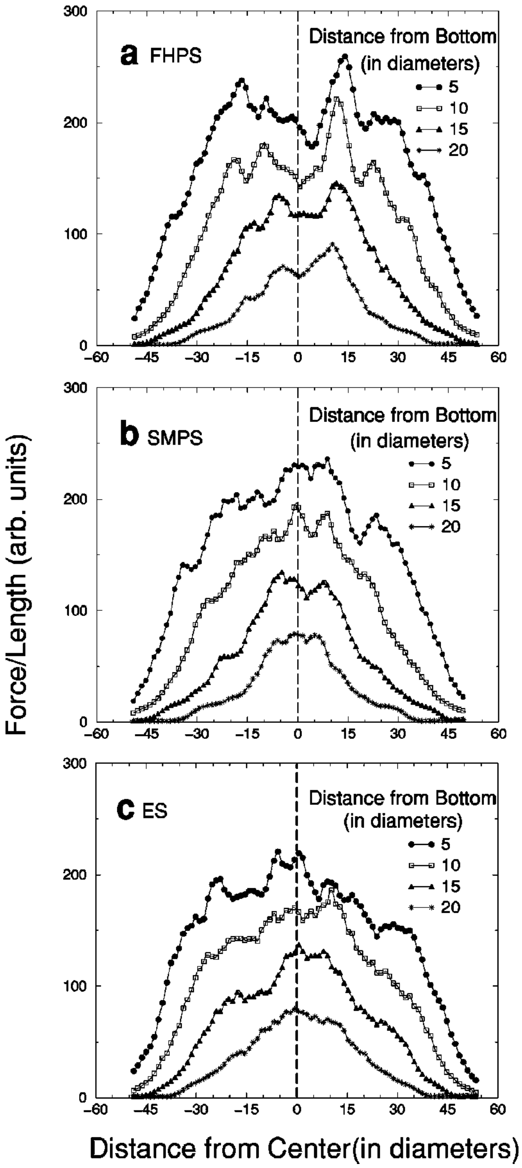

FIG. 3. Force as a function of horizontal distance along the indicated vertical distances from the bottom of the heap for three different pouring techniques: (a) fixed-height point source (FHPS); (b) slowly-moving point source (SMPS); (c) extended source (ES). The distances are measured in the diameter of the small particle.

of disks that experience a force exceeding the mean. These constitute $\sim 1/3$ of the disks. We show the corresponding $\rho(\theta)$ for the stress chain disks in Fig. 2(b) again only for the left half of the heap. In all cases, these large-force-carrying disks have contact angle distributions that break the roughly six-fold symmetry present in the distributions for all disks. This corresponds to stress chains that are inclined at angles $0<\theta<\pi/2$, i.e., in such a way as to support the heap. This compares very well with the CD calculations of Moreau [12].

The force profiles are affected by preparation history, but not as dramatically as the contact angles. In Fig. 3 we show the force (averaged over 50 samples) as a function of horizontal distance for various heights from the bottom of the heap, measured in small particle diameters. The fixed-height

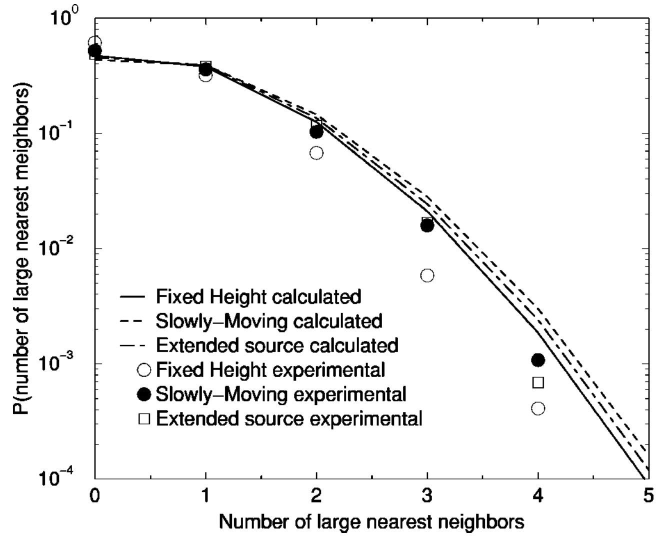

FIG. 4. Probability distributions for the number of large nearest neighbors for three different pouring techniques: fixed-height point source (FHPS), slowly-moving point source (SMPS), and extended source (ES). Points are experimental data and the curves are calculated as described in the text.

local source method clearly yields a force minimum in the center of the heap. The slowly lifted point source does not show a convincing dip, at least within the scatter of the data. Similarly, the extended source method shows no dip.

A final consideration is particle segregation, since significant segregation might lead to a pressure dip [11]. In our experiments, we did not observe the strong segregation assumed in DEM calculations [11]. This point is documented in Fig. 4, where we compare experimental distribution data (points) and calculated random distributions (lines) for the probability $P(n_l)$ of finding n_l large particles next to any given particle. We estimate $P(n_l)$ as $P(n_l)=\Sigma_{n=2}^{6}\alpha_n P_{n,n_l}$, where P_{n,n_l} is the binomial distribution for a total of $n=n_s+n_l$ contacts (n_s the total number of contacts with small particles), and where α_n is fraction of particles in the sample with coordination number n. We determine the α_n separately for each construction technique, hence the different lines in Fig. 4. There is no indication of significant segregation, which may be because the particles remain in a relatively dense state, which limits the freedom of particles to segregate.

To conclude, these experiments clearly show that different methods of heap preparation lead to dramatically different distributions of contact angles, hence, the texture, for physical granular systems. Knowledge of this texture is then crucial for predicting static granular states. Although we do not determine information on the vector forces at frictional contacts, it is important to emphasize that the most important difference between the various filling techniques is the amount of disorder in the contact orientations. This does not seem to depend on segregation effects.

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