Problems:

1. Deduce the virial expansion

\[ \frac{PV}{nkT} = \sum_{l=1}^{\infty} a_l \left( \frac{\lambda^3}{v} \right)^{l-1} \]

using the expressions:

\[ \frac{P}{kT} = \frac{1}{\lambda^3} g_{5/2}(z) \]

and

\[ \frac{N - N_0}{V} = \frac{1}{\lambda^3} g_{3/2}(z) \]

with \( \lambda \) the thermal wavelength and \( g(z) \) the Bose Einstein functions. Verify the quoted values of the virial coefficients – it is sufficient to find the first three terms, \( a_1, a_2, \) and \( a_3 \). [30 points]

2. Consider an ideal Bose gas in the grand canonical ensemble and study fluctuations in the total number of particles \( N \) and total energy \( E \). Discuss, in particular, the situation when the gas becomes highly degenerate. Use the following definitions:

\[ \langle (N - \langle N \rangle)^2 \rangle = kT \left( \frac{\partial \langle N \rangle}{\partial \mu} \right)_{T,V} \equiv (\Delta N)^2 \]

\[ \langle (E - \langle E \rangle)^2 \rangle = kT^2 \left( \frac{\partial U}{\partial T} \right)_{z,V} \equiv (\Delta E)^2 \]

[20 points]