Assignment #5
Due at the beginning of class on Wednesday, 3/01/06

PROBLEMS:

1. A Loaded Die: [25 points]
   On a trip to Las Vegas you visit a particularly seedy casino. You observe that at one of
   the gambling tables a die is being used which shows the number 6 twice as frequently
   as the number 1. Nothing unusual is noticed about the frequency of the other faces.
   Using the criterion of maximum entropy, what are the probabilities
   \( p_m \) (1 \( \leq m \leq 6 \)) one
   should assign to the various faces? (hint: use the method of Lagrange multipliers...)

2. Paramagnet in the Microcanonical Ensemble: [25 points]
   Consider a system of \( N \) magnetic dipoles in the microcanonical ensemble. Enumerate
   the number of microstates \( \Omega(N, E) \), accessible to the system at energy \( E \), and evaluate
   the quantities \( S(N, E) \) and \( T(N, E) \). Compare your results with those derived in class
   for the canonical ensemble (Pathria, eqns. 3.10.9 and 3.10.8).

3. Equipartition for Toy Molecules
   (a) Consider a “molecule” consisting of two classical point particles of mass \( m \) joined
   by a spring with stiffness \( k \). Use the equipartition theorem to determine the
   internal energy of a gas of \( N \) such molecules as a function of temperature. [10
   points]
   (b) Repeat part (a) for molecules consisting of three point particles joined by three
   springs forming an equilateral triangle. [10 points]

4. Show that the entropy of a system in the grand canonical ensemble can be written as
   \[ S = -k \sum_{r,s} P_{r,s} \ln P_{r,s} \]
   where \( P_{r,s} \) is given by the grand canonical distribution
   \[ P_{r,s} = \frac{\exp(-\mu N_r/(kT) - E_s/(kT))}{\sum_{r,s} \exp(-\mu N_r/(kT) - E_s/(kT))} \].
   [30 points]