Assignment #3
Due at the beginning of class on Wednesday, 2/16/06

Problems:

1. Two walkers living in a one-dimensional world begin at the same location. On each time step, each walker chooses a random direction, left or right, and takes a step of length 1 unit. What is the probability that the two walkers will be at the same place after $N$ steps? [10 points]

2. The meaning of Never
Suppose $10^{10}$ monkeys sit at typewriters. Every monkey hits 10 keys per second on a typewriter with 25 keys (no distinction between upper case and lower case letters).

(a) What is the probability for a monkey to reproduce by chance a given sequence of $N = 5$ letters? How long will this take (on average)? [5 points]

(b) What is the probability that any sequence of $N = 10^5$ characters will match Shakespeare’s *Hamlet* which has approximately $10^5$ characters? [5 points]

(c) What is the probability that the correct *Hamlet* will be typed by any of the monkeys during the age of the Universe (which is approx. $10^{18}$ seconds)? [5 points]

3. Consider an isolated system consisting of a large number $N$ of very weakly interacting localized particles of spin $\frac{1}{2}$. Each particle has a magnetic moment $\mu$ which can point either parallel or antiparallel to an applied field $H$. The energy $E$ of the system is then $E = -(n_1 - n_2)\mu H$, where $n_1$ is the number of spins aligned parallel to $H$ and $n_2$ the number aligned antiparallel to $H$.

(a) Consider the energy range between $E$ and $E + \Delta$, where $\mu H << \Delta << E$. What is the total number of states $\Omega(E)$ lying in this energy range? [5 points]

(b) Write an expression for $\ln \Omega$ as a function of $E$ and simplify it using Stirling’s formula. [5 points]

(c) For $E$ not close to its extreme values of $\pm N\mu H$, express $\Omega(E)$ as a Gaussian function of $E$. [5 points]

4. (a) Derive an asymptotic expression for the number of ways in which a given energy $E$ can be distributed among a set of $N$ one-dimensional harmonic oscillators, the energy eigenvalues of the oscillators being $(N + 1/2)\hbar \omega$; $n = 0, 1, 2, \ldots$ You may assume that the individual oscillators are distinguishable; e.g., they are located at different positions in space. [5 points]

(b) Derive the corresponding expression for the it volume of the relevant region of the phase space of this system. Establish the correspondence between the two results, showing that the conversion factor $\omega_0$ is precisely $\hbar^N$. [15 points]