READING: Review the fundamentals of thermodynamics using a familiar textbook, e.g. Greiner, Stöcker & Neise chapters 1, 2 and 4.

PROBLEMS:

1. Write defining relations for the following quantities and give the SI units of each quantity. Define each of the symbols that appears.
   (a) heat capacity [2 points];
   (b) isothermal compressibility [2 points];
   (c) coefficient of thermal expansion [2 points];
   (d) thermal conductivity [2 points];
   (e) viscosity (of a gas) [2 points].

2. Adiabatic Compression: A diesel engine requires no spark plug. Rather, the air in the cylinder is compressed so highly that the fuel ignites spontaneously when sprayed into the cylinder.
   (a) If the air is initially at room temperature (taken as 20 °C) and is then compressed adiabatically by a factor of 15, what final temperature is attained (before fuel injection)? For air, the ratio of heat capacities is $\gamma = 1.4$ in the relevant range of temperature [5 points].
   (b) By what factor does the pressure increase [5 points]?

3. In Mayer’s cycle, a mole of an ideal gas undergoes the following 3 steps. The gas begins with pressure $P_1$ and volume $V_1$. In step 1, it is allowed to expand adiabatically to volume $V_2$, at which point the pressure is $P_2$. In step 2, it is quasi-statically compressed back to $V_1$ while the pressure is maintained at $P_2$. Finally, in step 3, the gas is heated quasi-statically at fixed volume until the pressure returns to $P_1$.
   (a) Draw a $P$-$V$ diagram representing Mayer’s cycle [4 points].
   (b) For each step $i$, calculate $W_i$ (the work done on the gas) and $Q_i$ (the heat absorbed by the gas). Express your results in terms of the molar specific heats, $c_v$ and $c_p$ (assumed constant), and the gas constant $R$ [10 points].
   (c) Using the fact that the gas returns to its original state after one cycle, show that
   \[ c_p = c_v + R. \] [6 points]

4. The speed of sound, $c$, in a gas is given by the thermodynamic derivative
\[
 c^2 = \frac{1}{m} \left( \frac{\partial P}{\partial n} \right)_{S,N}
\]
where $m$ is the mass of a gas particle and $n$ is the density $N/V$. Express $c$ in terms of the heat capacity, the coefficient of thermal expansion $\beta$, and the isothermal compressibility $\kappa_T$ [30 points].
5. Consider a gas of $N$ particles of mass $m$ confined to a volume $V$. The Helmholtz free energy of the gas is

$$F(T, V, N) = NkT \ln \left[ \frac{N}{e} \frac{k^3}{(V - Nb)(2\pi mkT)^{3/2}} \right] - \frac{N^2}{V} a,$$

where $T$ is the temperature, $a$ and $b$ are positive constants and $e = 2.718\ldots$.

(a) Show that this gas obeys the van der Waals equation of state,

$$\left( P + \frac{a}{v^2} \right)(v - b) = kT \quad \text{with: } v = \frac{V}{N}$$

[10 points]

(b) Derive the relation between temperature and volume of this gas when it undergoes an adiabatic process [10 points].

(c) Find the change in temperature $\Delta T$ when the gas makes a free expansion from $V$ to $V + \Delta V$ [10 points].

HINTS:

1. For problems 4 and 5, use the following relation:

$$\left( \frac{\partial x}{\partial y} \right)_z \cdot \left( \frac{\partial y}{\partial z} \right)_x \cdot \left( \frac{\partial z}{\partial x} \right)_y = -1$$

2. In problem 5c, Maxwell’s relations might be useful.