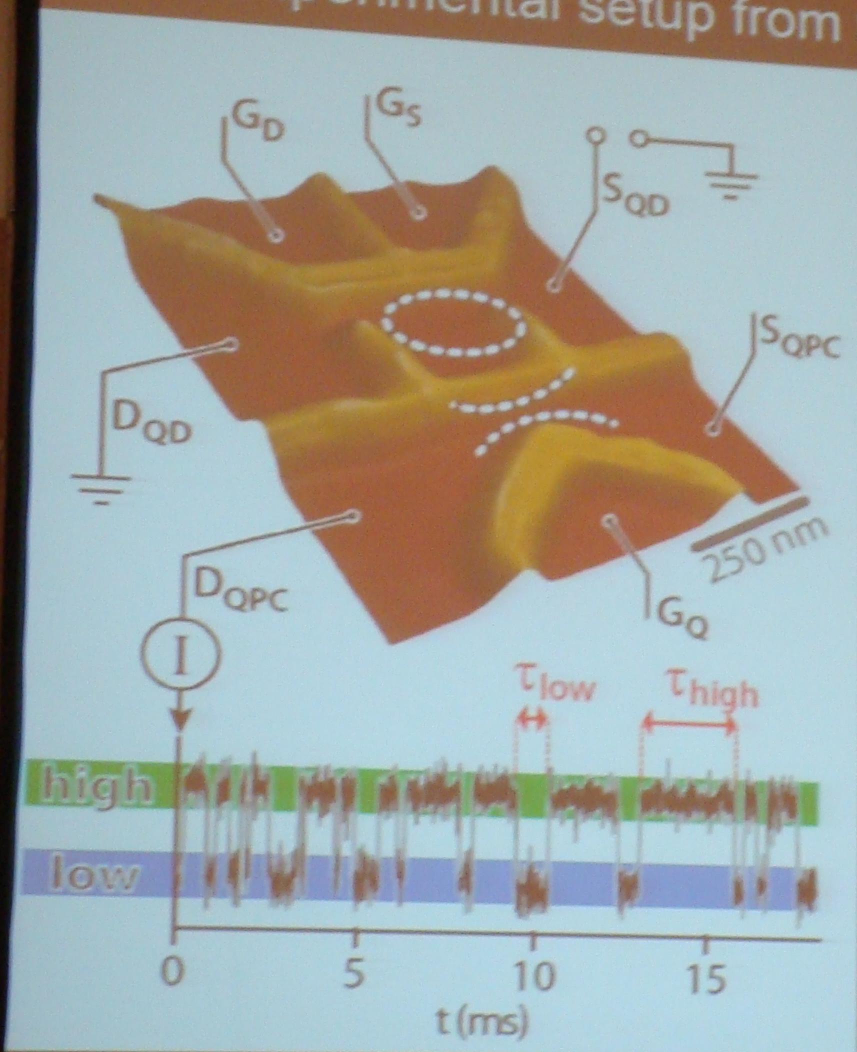
High-order cumulants

Experimental setup from the Haug group (Hannover)



- Single electron tunneling through Coulomb blockade quantum dot (QD)
- QPC current sensitive to QD occupatio
- Similar experiments at ETH-Zürich
  See talk by T. Ihn (X3.00002)
- Advantages:
  - tunable tunneling rates
  - single-electron detection
  - high-quality statistics
- Draw-backs:
  - slow tunneling rates (~ kHz regime)

C. Flindt, C. Fricke, F. Hohls, T. Novotný, K. Netocný, T. Brandes, and R. J. Haug

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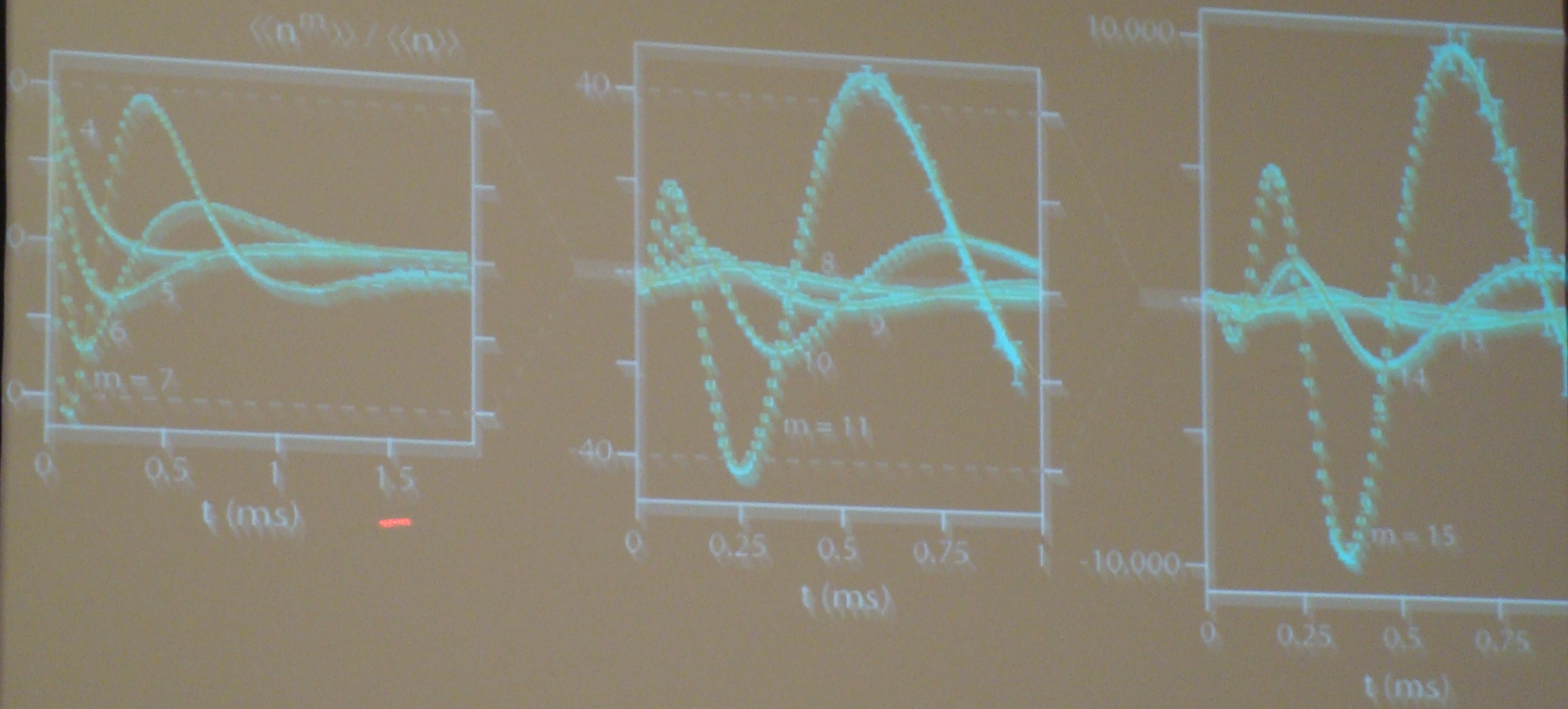


- The cumulants oscillate with time and grow dramatically with the order!
- Similar results found theoretically in several different systems
- What is the origin of this behavior?

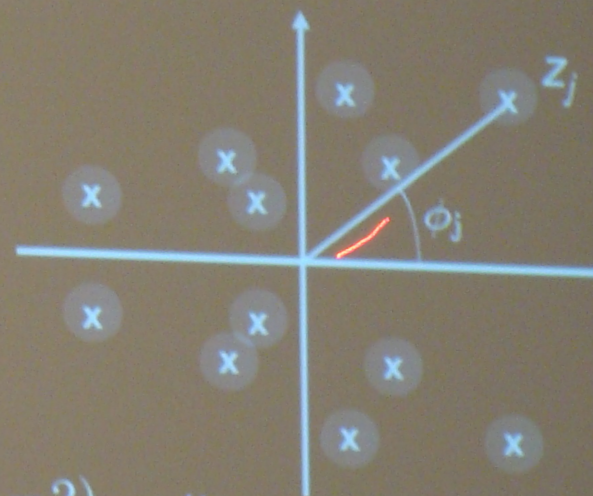
igh-order cumulants

- Cumulants are *derivatives* of the cumulant generating function (CGF)
- High-order derivatives of a function are determined by its singularities in the complex plane [M. V. Berry, Proc. R. Soc. A **461** (2005) 1735]

- Close to a singularity z_j:

$$\mathcal{S}(z) \simeq A_j/(z-z_j)^{\mu_j}$$ for some A_j and μ_j.

$$\partial_z^m \mathcal{S}(z) \simeq \frac{(-1)^{\mu_j} A_j B_{m,\mu_j}}{|z-z_j|^{m+\mu_j}} e^{-i(m+\mu_j)\phi_j}$$



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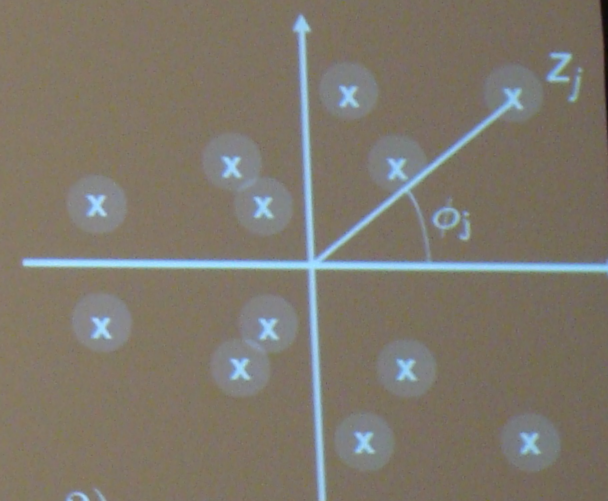

$$\langle\langle n^m \rangle\rangle = \partial_z^m \mathcal{S}(z)|_{z\to 0} \simeq \sum_j \frac{(-1)^{\mu_j} A_j B_{m,\mu_j}}{|z_j|^{m+\mu_j}} e^{-i(m+\mu_j)\phi_j}$$

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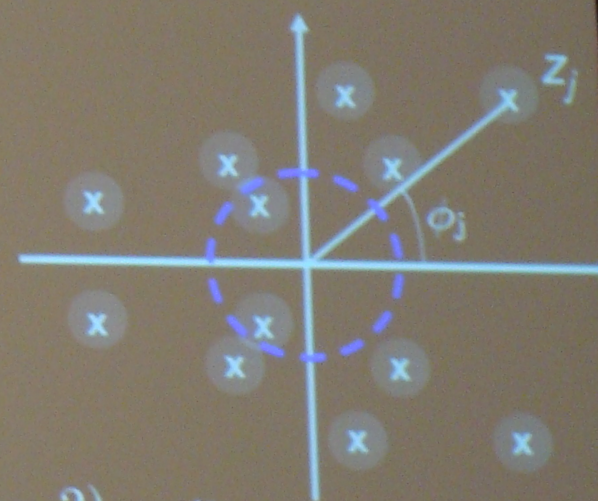

$$\partial_z^m \mathcal{S}(z) \simeq \frac{(-1)^{\mu_j} A_j B_{m,\mu_j}}{\ldots} e^{-i(m+\mu_j)\phi_j}$$

Singularities closest to 0 dominate the sum

$$\ldots + \mu_j - 2) \ldots \mu_j$$

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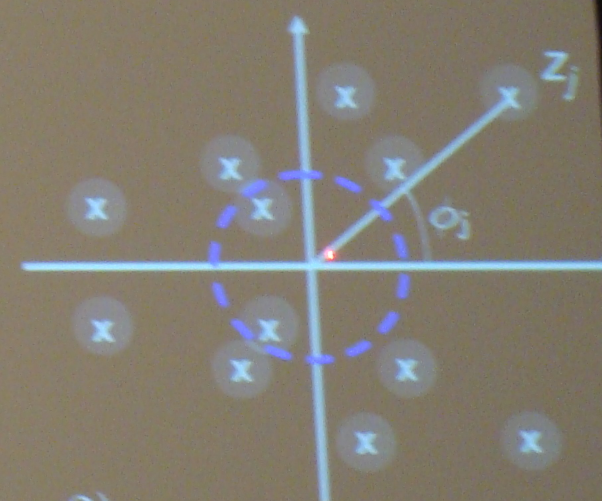

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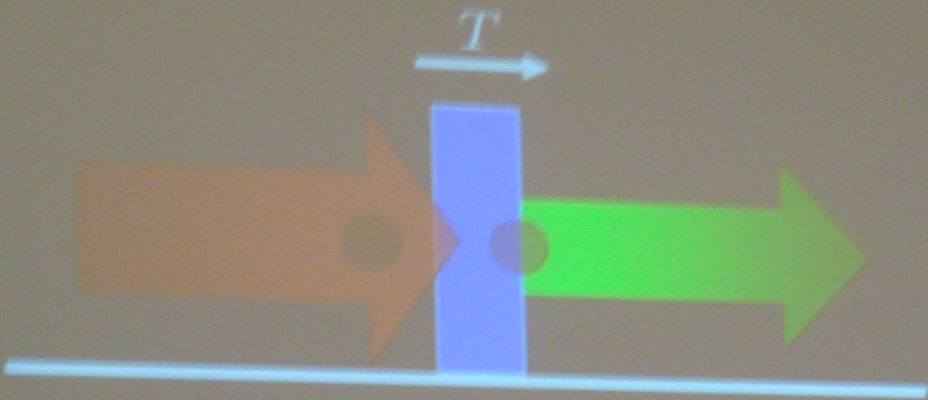
Cumulants oscillate as functions of parameters that change φ_j

$$\langle\langle n^m \rangle\rangle = \partial_z^m \mathcal{S}(z)|_{z\to 0} \simeq \sum_j \frac{(-1)^{\mu_j} A_j B_{m,\mu_j}}{|z_j|^{m+\mu_j}} e^{-i(m+\mu_j)\phi_j}$$

High-order cumulants

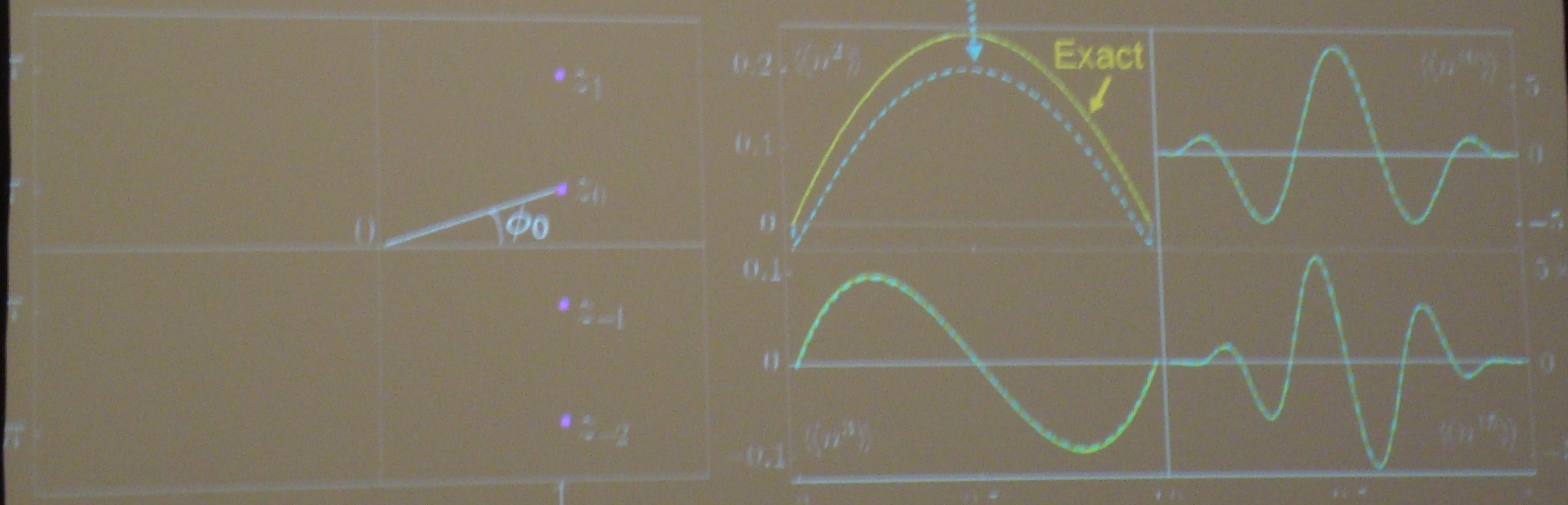


Simple "one-shot" example:

One charge with probability T of being transmitted

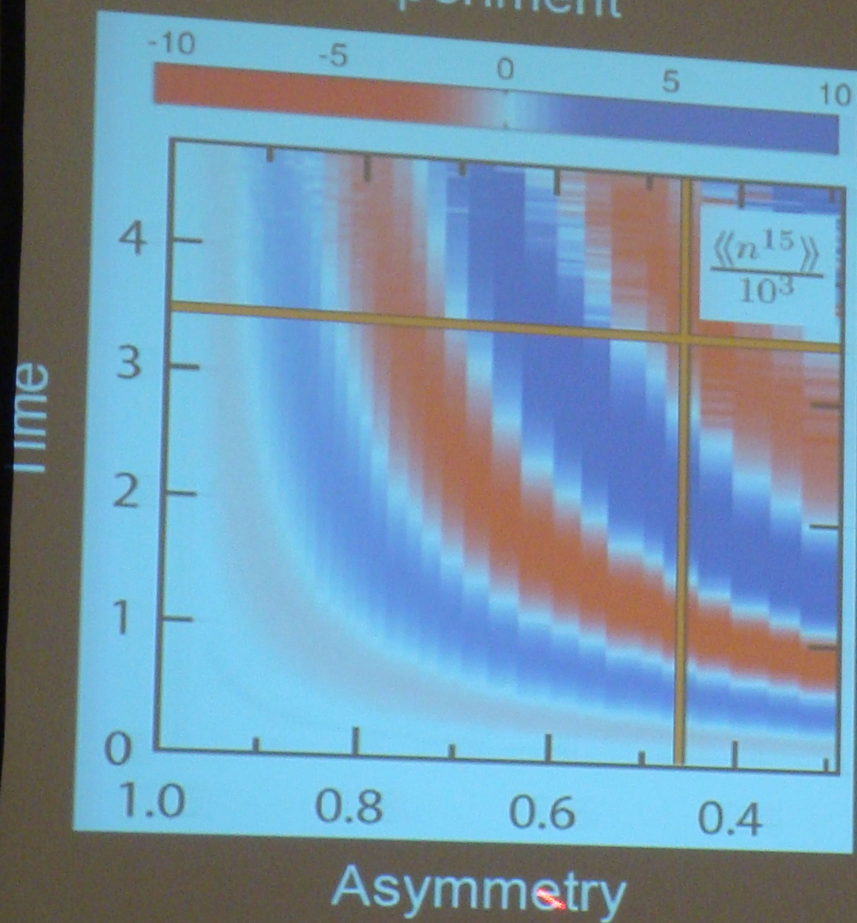
$$\mathcal{S}(z) = \ln[(1-T) + Te^{z}]$$

$$\langle\langle n^m \rangle\rangle \simeq -\frac{2(m-1)!}{|z_0|^m}\cos(m\phi_0)$$

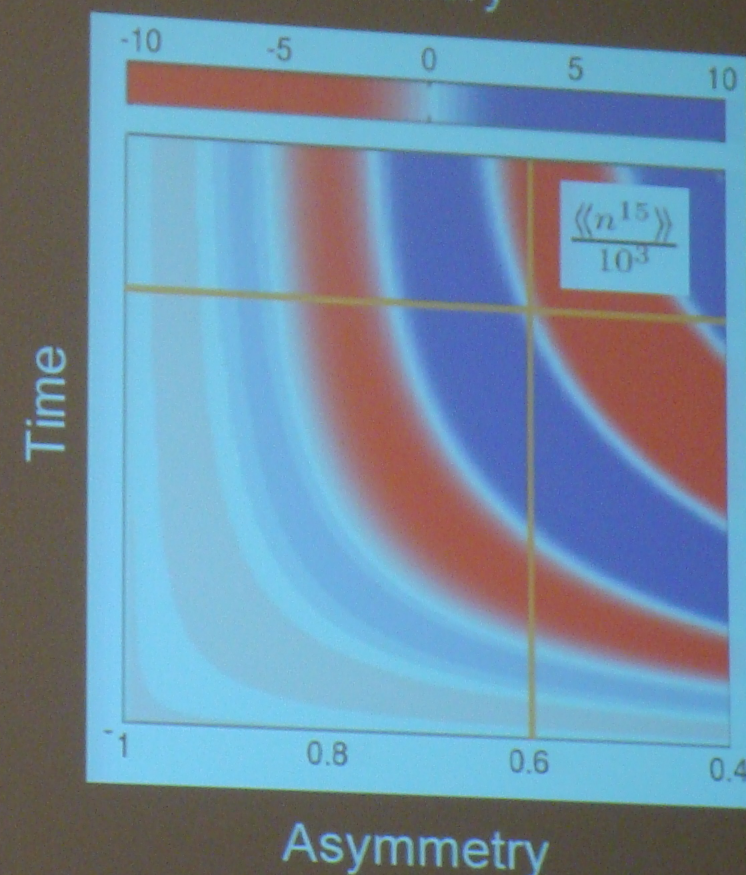


High-order cumulants

Experiment



Theory



Experiment: C. Fricke *et al.*, Appl. Phys. Lett. **96**, 202103 (2010)

Theory: D. Kambly, C. Flindt, and M. Büttiker, Phys. Rev. B **83**, 075432 (2011)

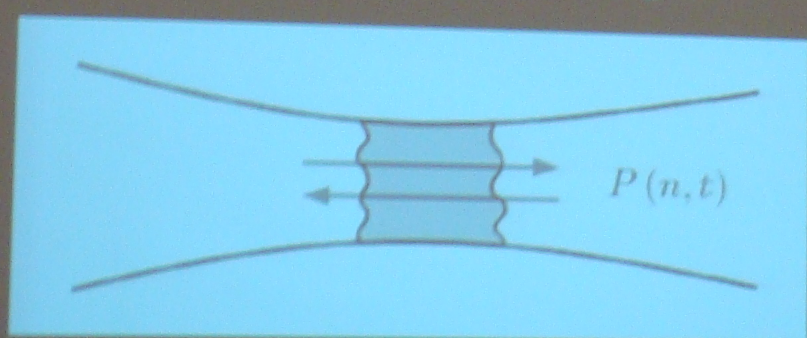
Factorial cumulants and interactions

The fact that these oscillations are so generic, does it imply that the high-order statistics don't contain any information?

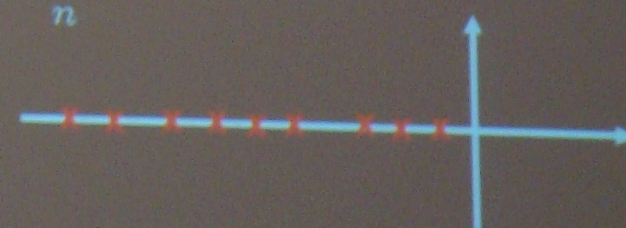
- The counting statistics for *non-interacting* electrons always consists of independent, binomial events (generalized binomial statistics).

  A. G. Abanov & D. A. Ivanov, PRL **100**, 086602 (2008), PRB **79**, 205315 (2009)
  F. Hassler *et al.* PRB **78**, 165330 (2008)

- The zeros of the generating function are correpondingly real and negative



$$\mathcal{G}(z,t) = \sum_n P(n,t) z^n$$



- Interactions can make the zeros move out into the complex plane

Factorial cumulants and interactions

- The position of the zeros can be determined using *factorial* cumulants!

$$\langle\langle n^m \rangle\rangle_F = \langle\langle n(n-1)\cdots(n-m+1)\rangle\rangle$$

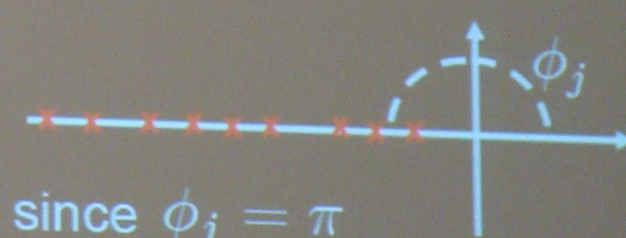
- Factorial cumulant generating function

$$\mathcal{S}_F(z,t) = \log \mathcal{G}(z+1,t) \;\Rightarrow \langle\langle n^m \rangle\rangle_F = \partial_z^m \mathcal{S}_F(z,t)|_{z\to 0}$$

- Zeros of $\mathcal{G}$ are logarithmic singularities of $\mathcal{S}_F$
- For *non-interacting* electrons the singularities are real and < -1.

- We then find:

$$\langle\langle n^m \rangle\rangle_F \simeq -\sum_j \frac{A_j(m-1)!}{|z_j|^m} \underbrace{e^{-im\phi_j}}_{=(-1)^m \text{ since } \phi_j = \pi}$$



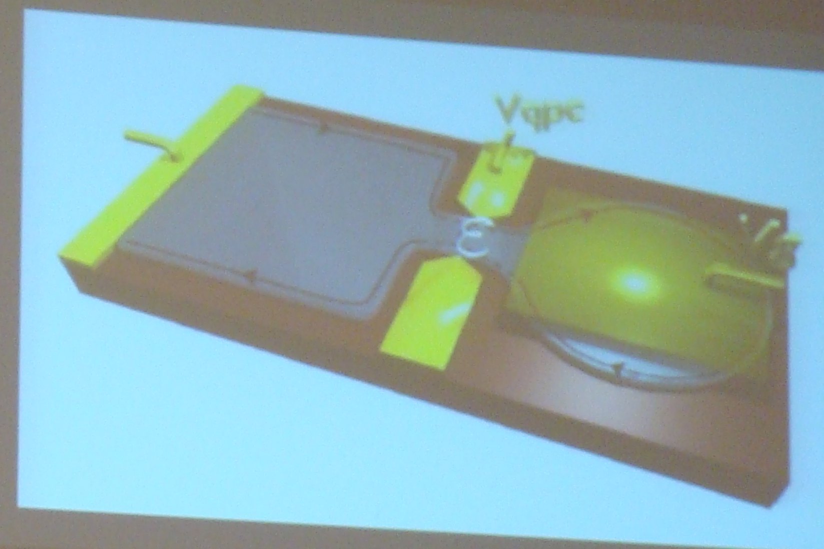
- High-order factorial cumulants *don't oscillate* for *non-interacting* electrons

Factorial cumulants and interactions

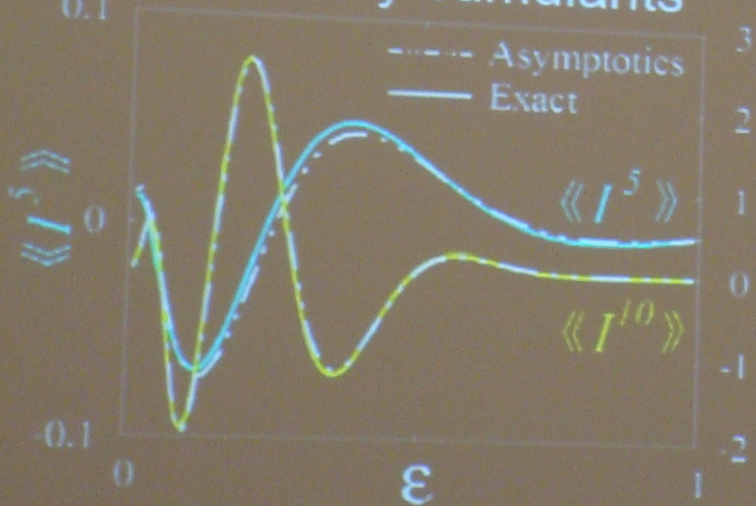
<u>Example: Non-interacting system</u>

AC-driven mesoscopic capacitor that emits *non-interacting* electrons into a chiral edge state

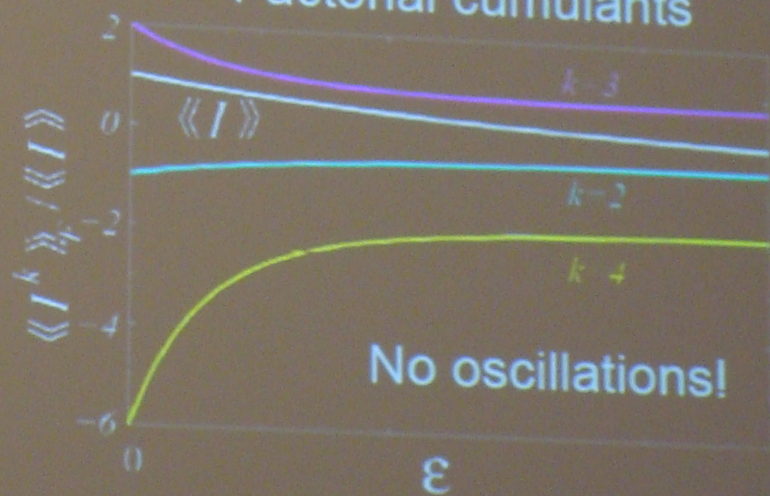
- M. Büttiker, H. Thomas, and A. Prêtre, Phys. Lett. A **180**, 364 (1993)
- G. Fève *et al.*, Science **316**, 1169 (2007)



Ordinary cumulants



Factorial cumulants



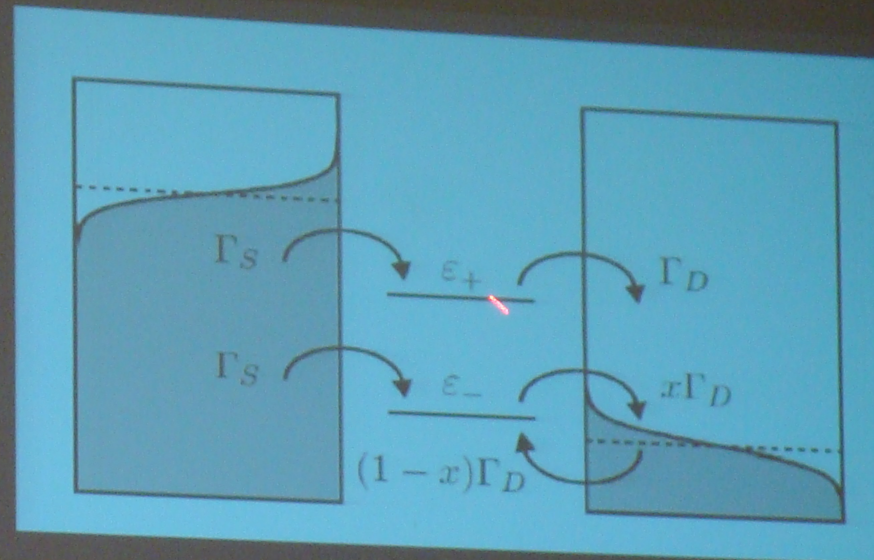
No oscillations!

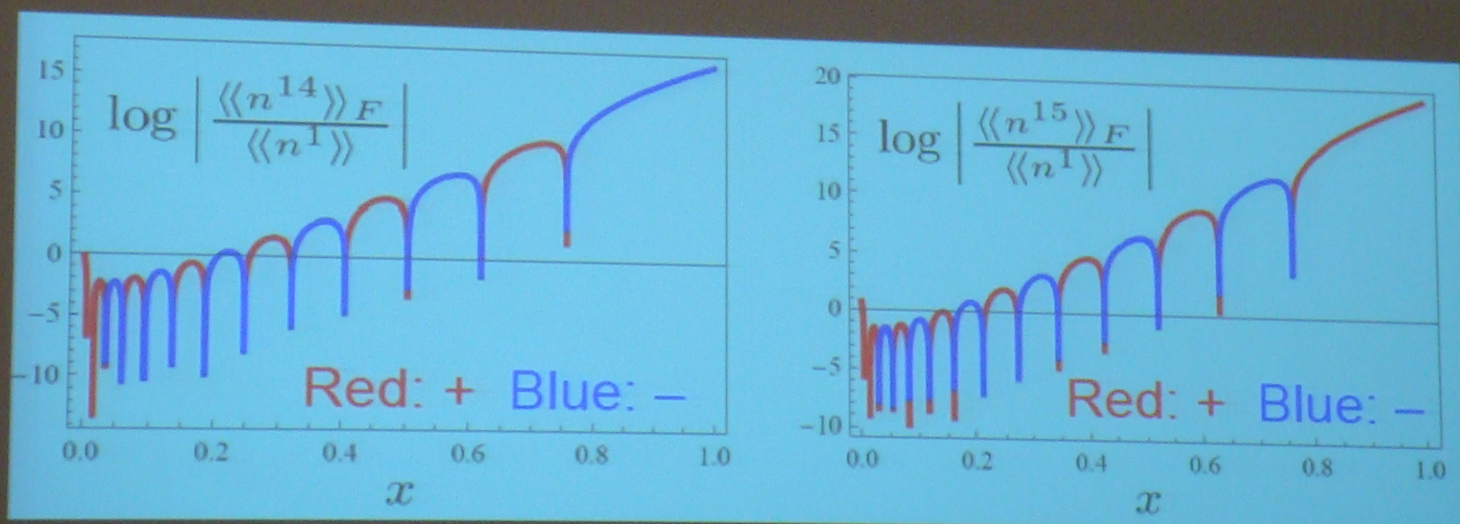
Factorial cumulants and interactions

<u>Example: Interacting system</u>

Two-level quantum dot, where only one level at a time can be occupied

- W. Belzig, Phys. Rev. B **71**, 161301R (2005)





Conclusions

- Cumulants oscillate as a function of basically any parameter
- Factorial cumulants, in contrast, don't oscillate for non-interacting electron
- Oscillating factorial cumulants are due to interactions

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Proc. Natl. Acad. Sci. USA **106**, 10116 (2009)

D. Kambly, C. Flindt, and M. Büttiker
Phys. Rev. **83**, 075432 (2011) – Editors' Suggestion

- Thanks to my collaborators

Thank you for your attention!

FNSNF

Swiss National Science Foundation