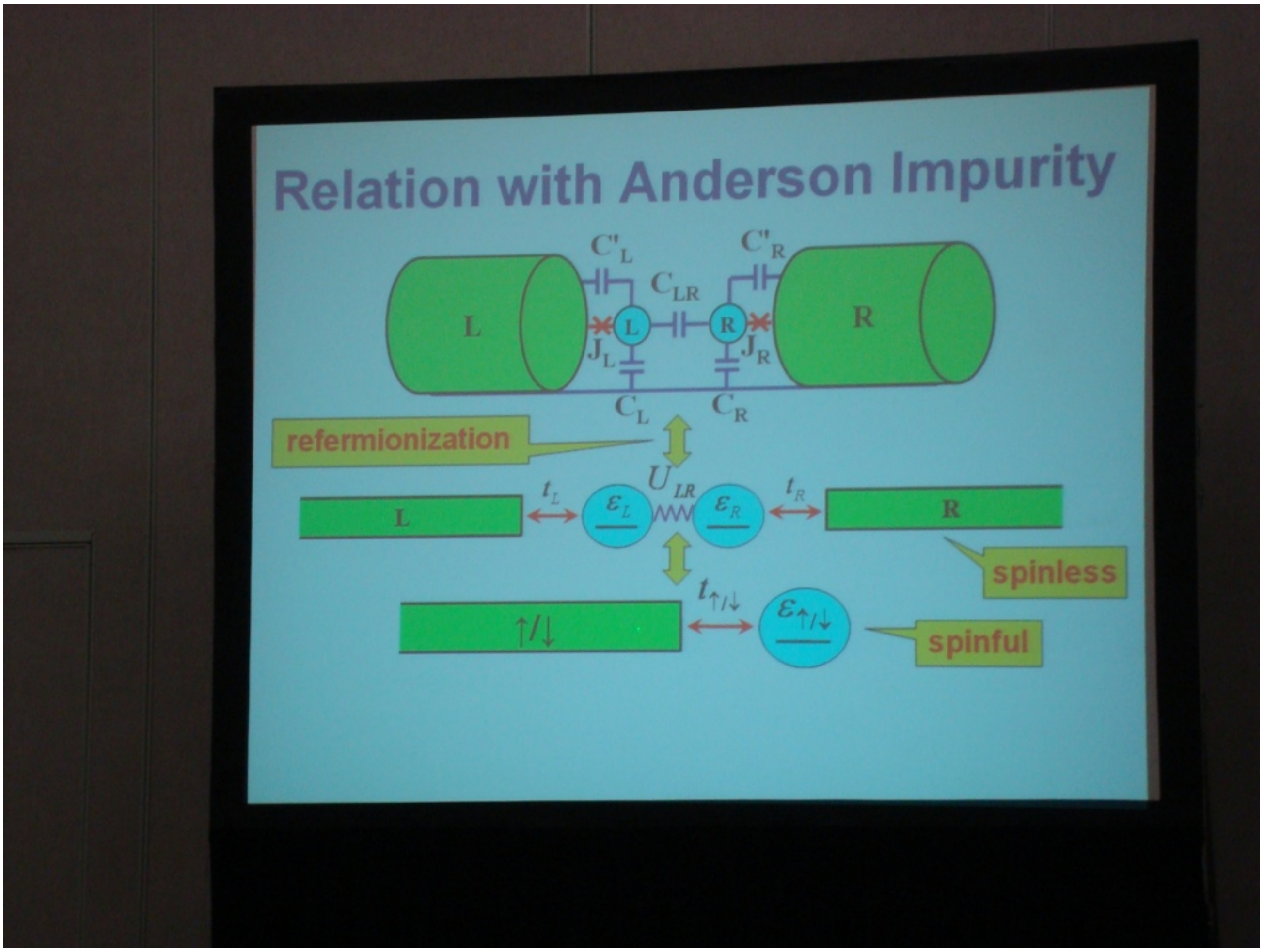
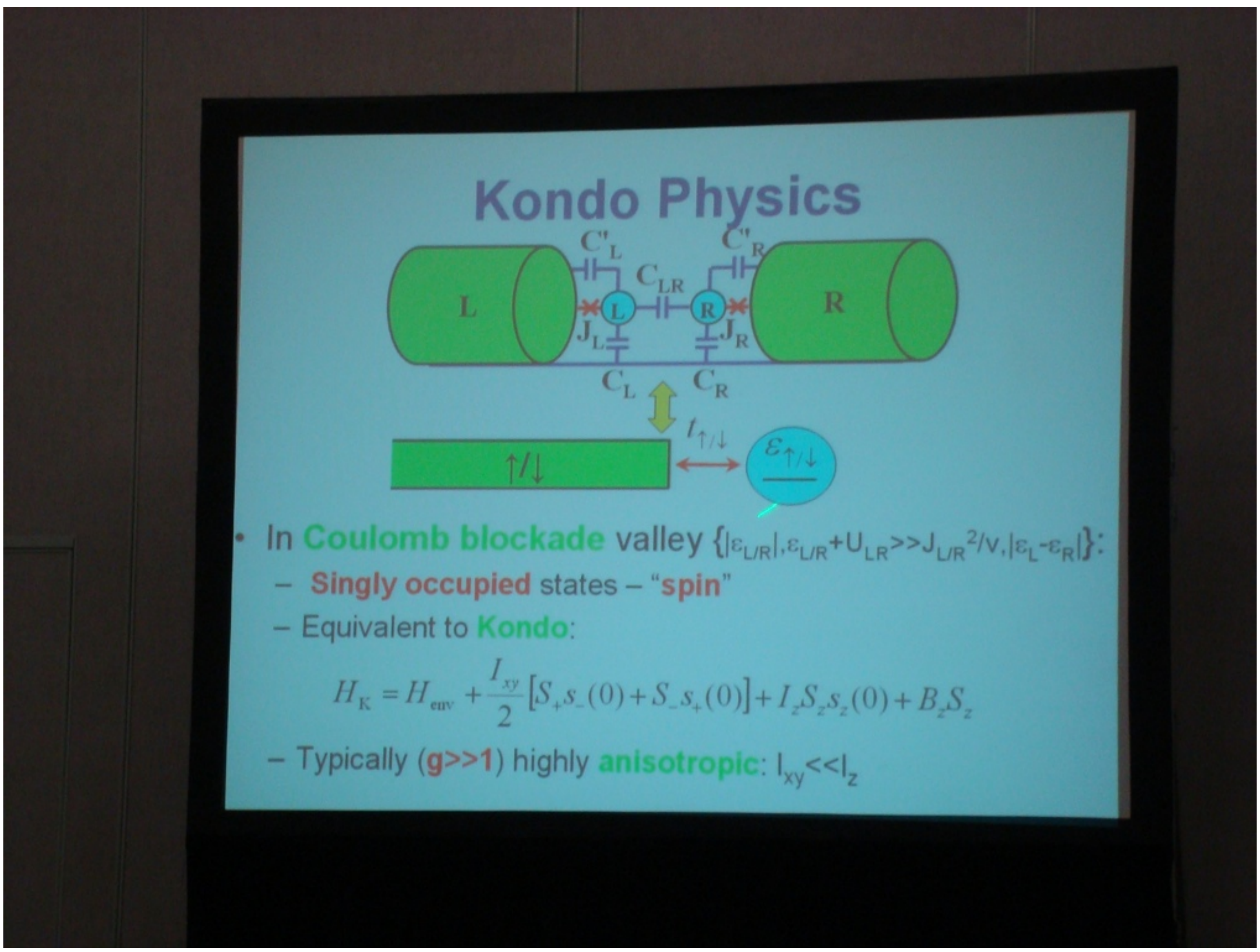
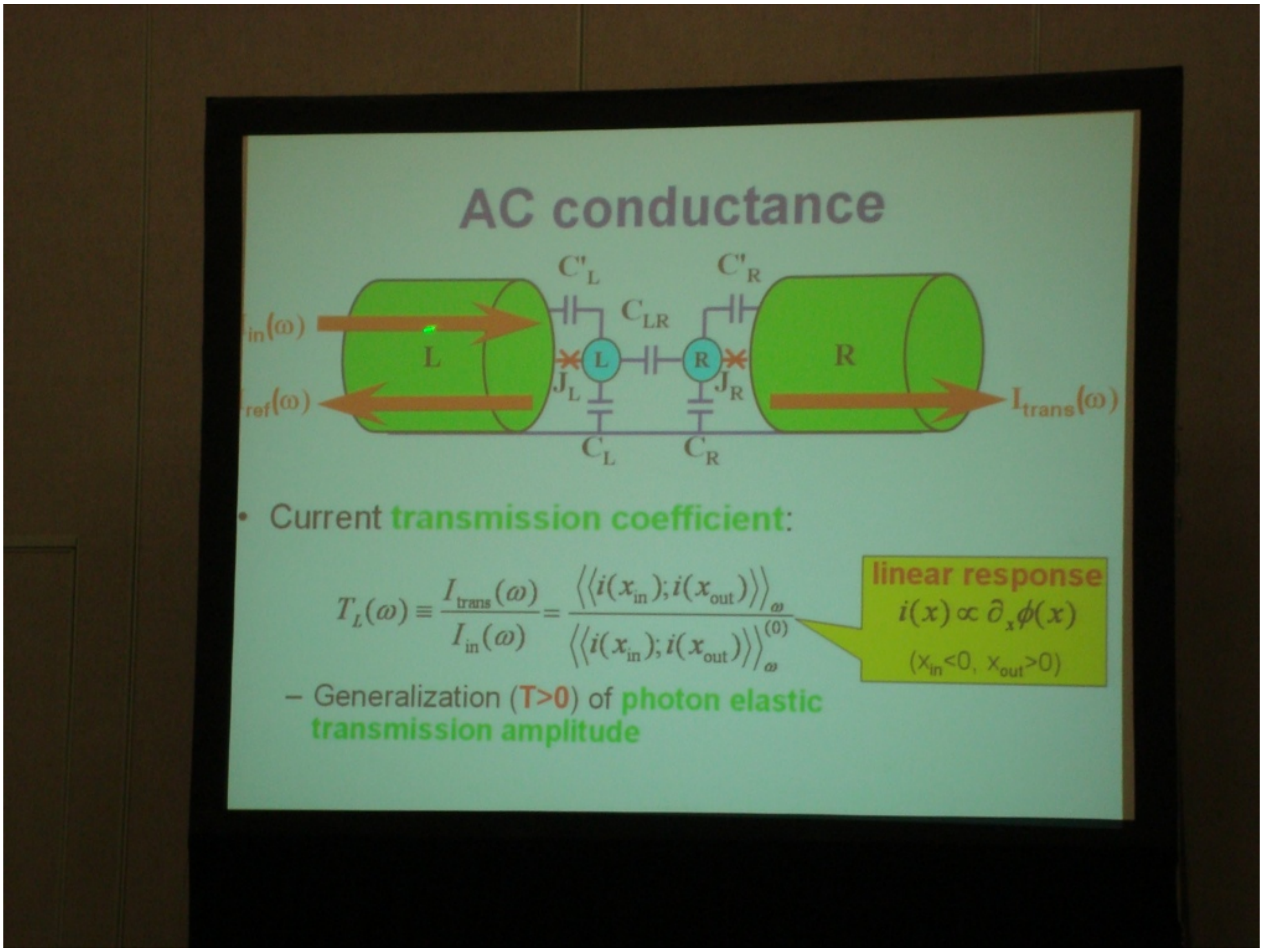
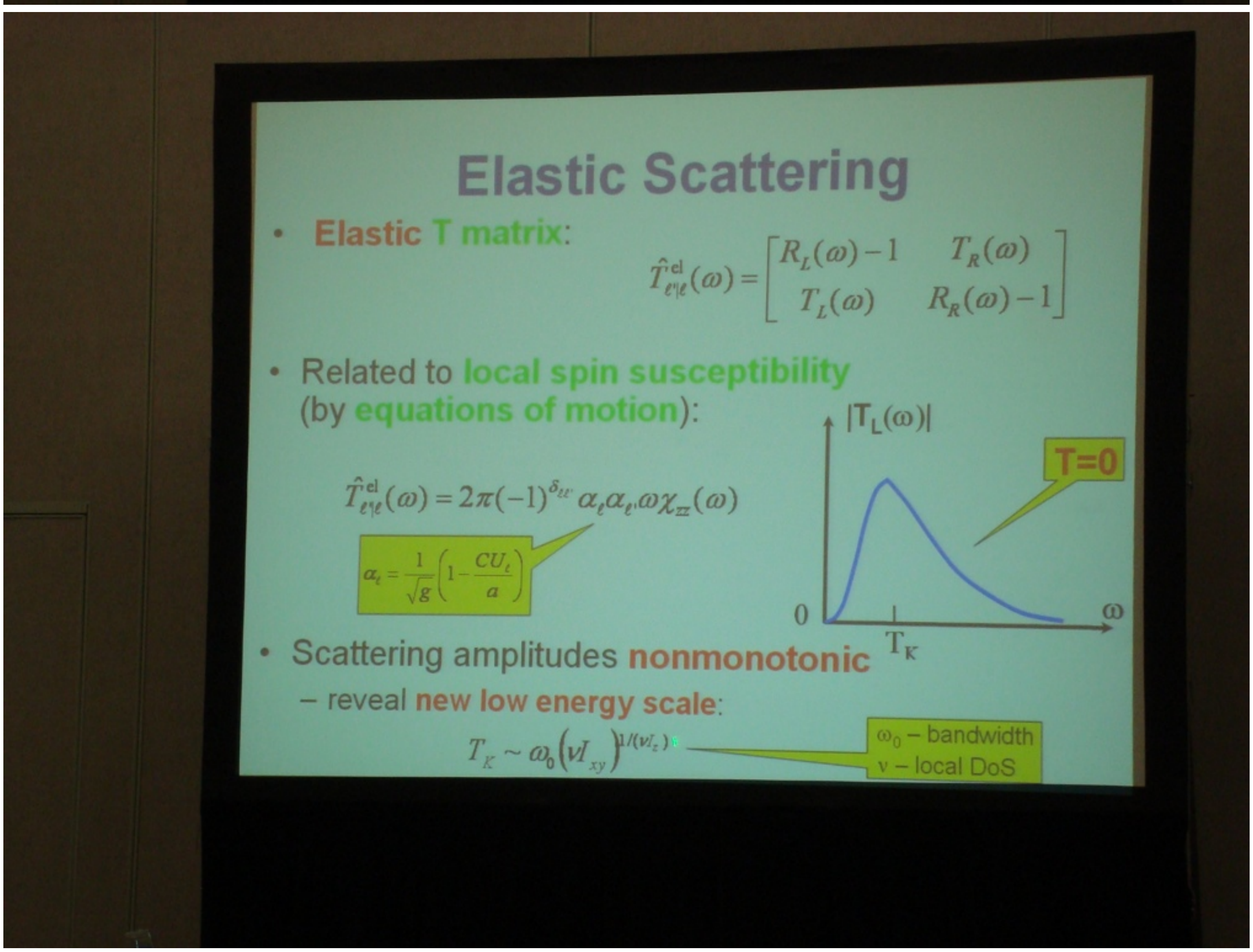
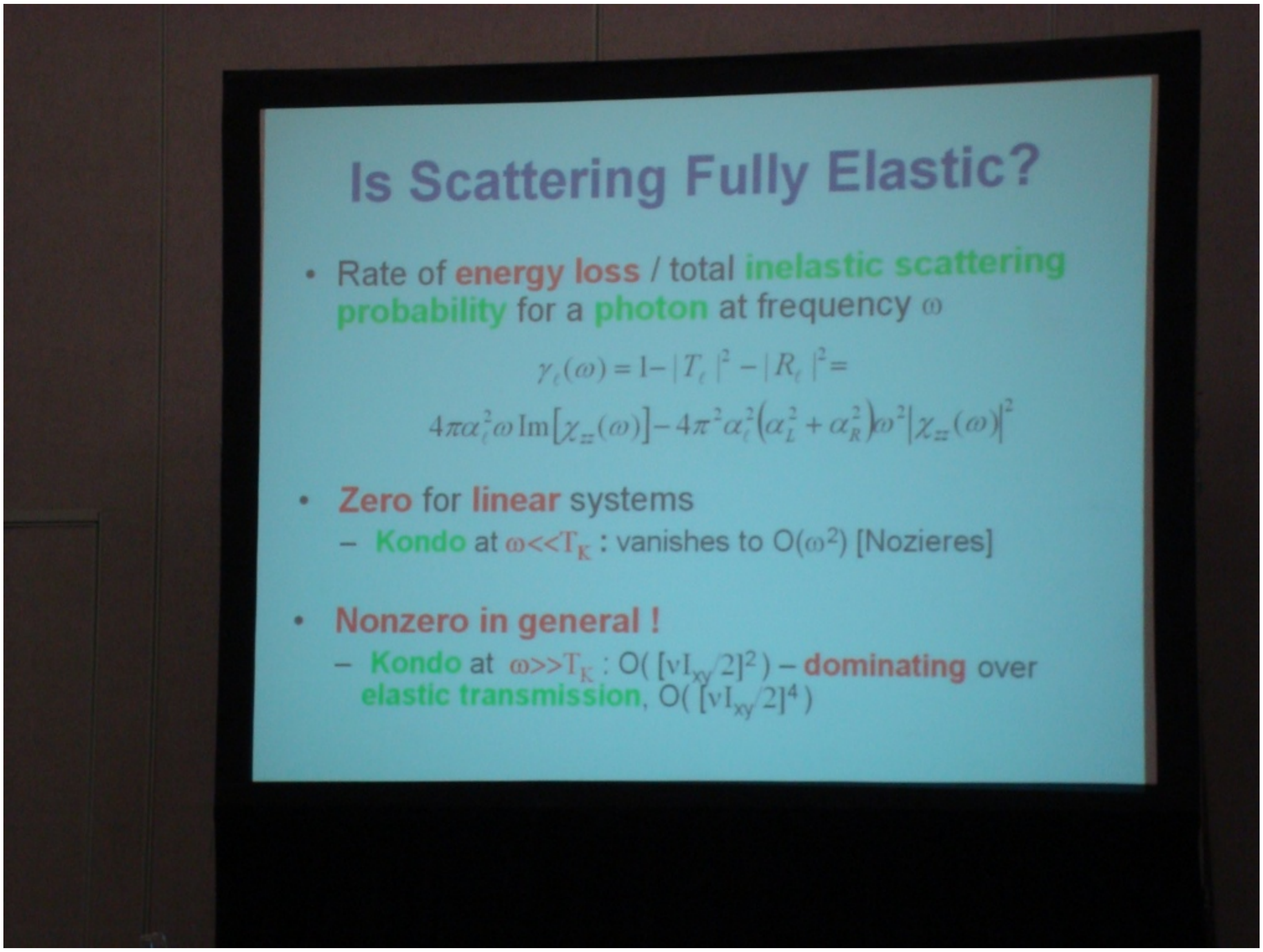
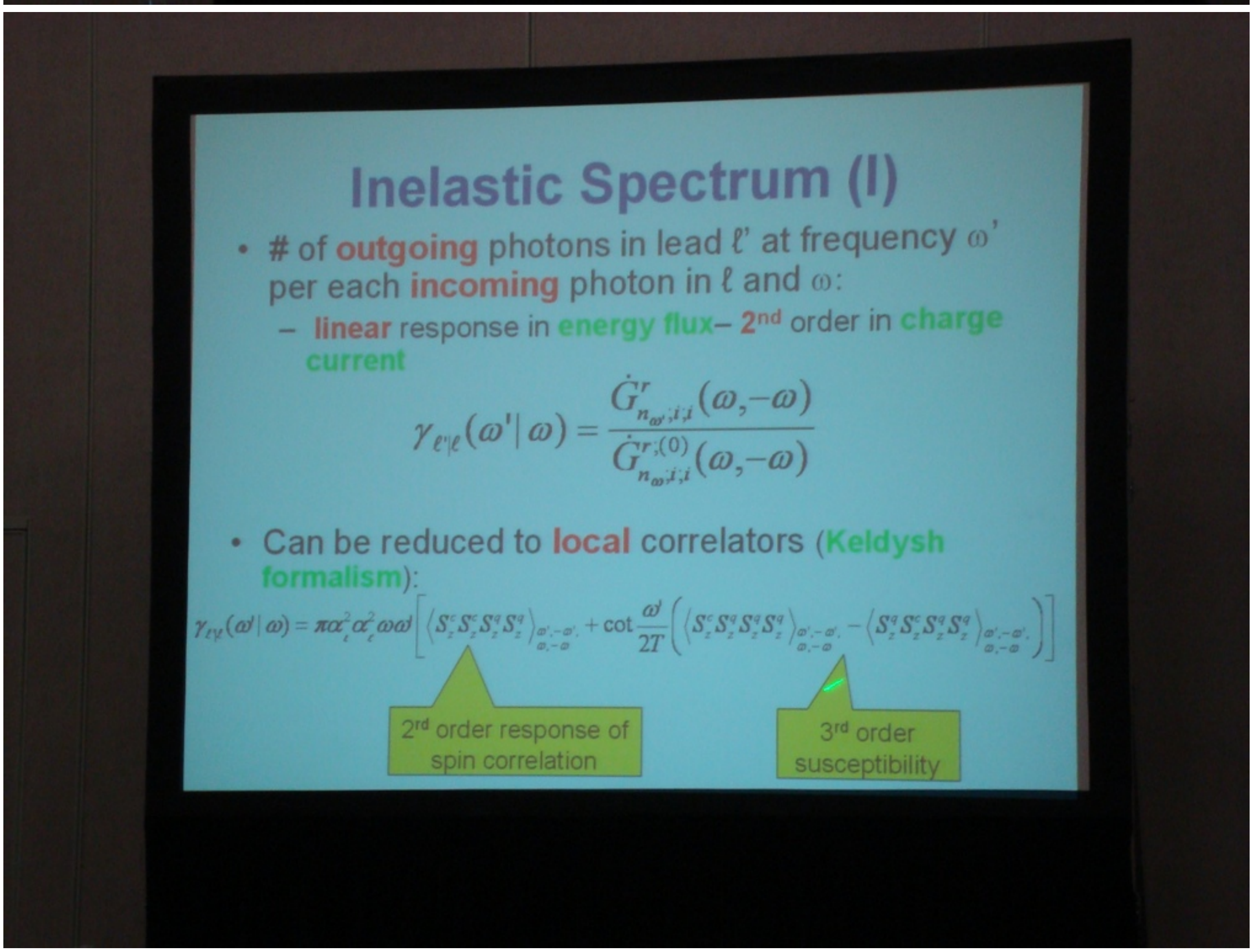

Relation with Anderson Impurity
C'L
C'R
CLR
L
R
JL
JR
CL
CR
refermionization
U_{LR}
t_L
t_R
ε_L
ε_R
spinless
$t_{\uparrow/\downarrow}$
$\uparrow/\downarrow$
$\varepsilon_{\uparrow/\downarrow}$
spinful

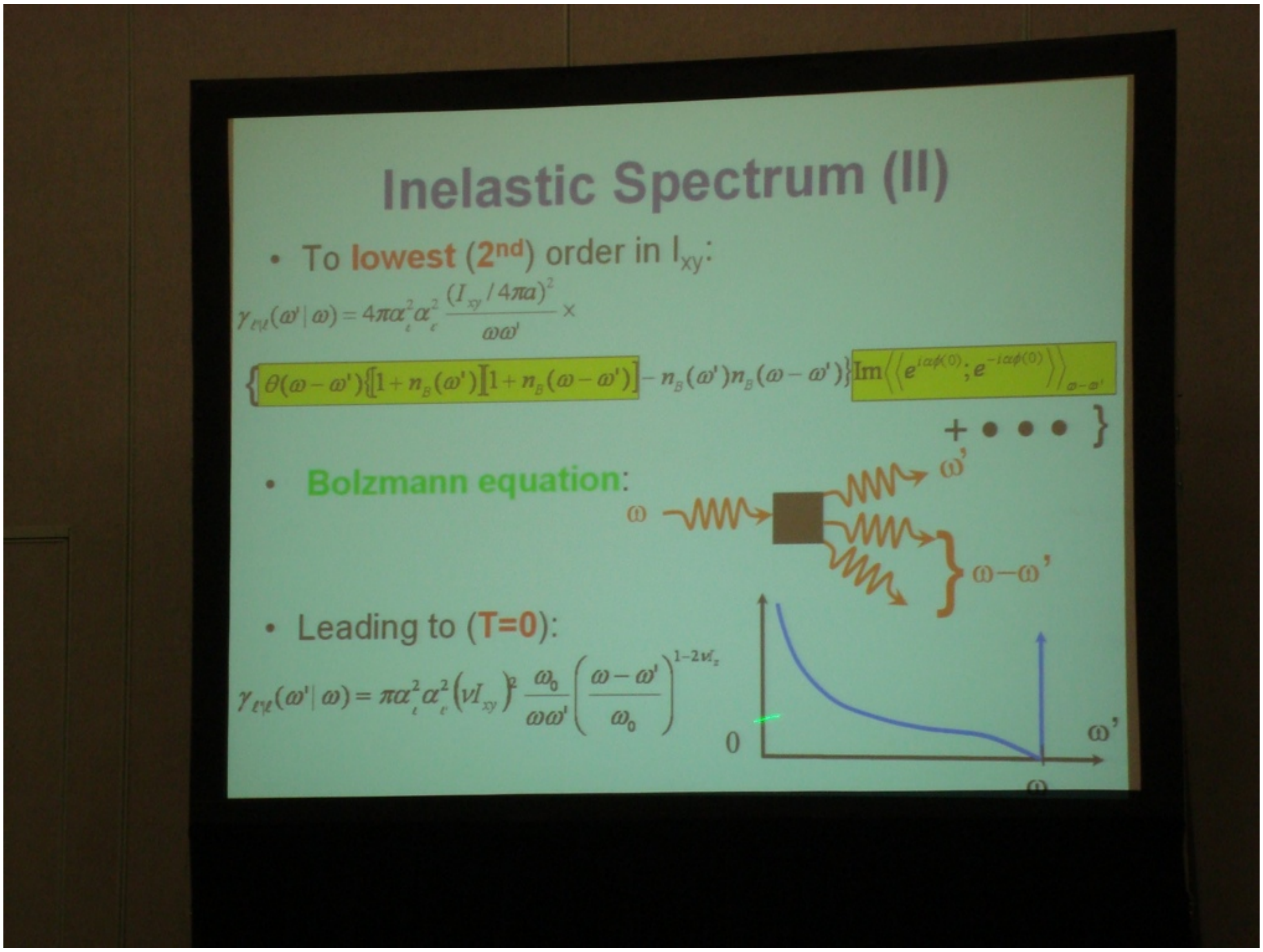

Kondo Physics
C'L
C'R
CLR
L
R
JL
JR
CL
CR
$t_{\uparrow/\downarrow}$
$\uparrow/\downarrow$
$\varepsilon_{\uparrow/\downarrow}$
• In Coulomb blockade valley $\{|\varepsilon_{L/R}|, \varepsilon_{L/R}+U_{LR} >> J_{L/R}{}^2/v, |\varepsilon_L-\varepsilon_R|\}$:
– Singly occupied states – "spin"
– Equivalent to Kondo:
$H_K = H_{env} + \frac{I_{xy}}{2}\left[S_+ s_-(0) + S_- s_+(0)\right] + I_z S_z s_z(0) + B_z S_z$
– Typically (g>>1) highly anisotropic: $I_{xy} << I_z$

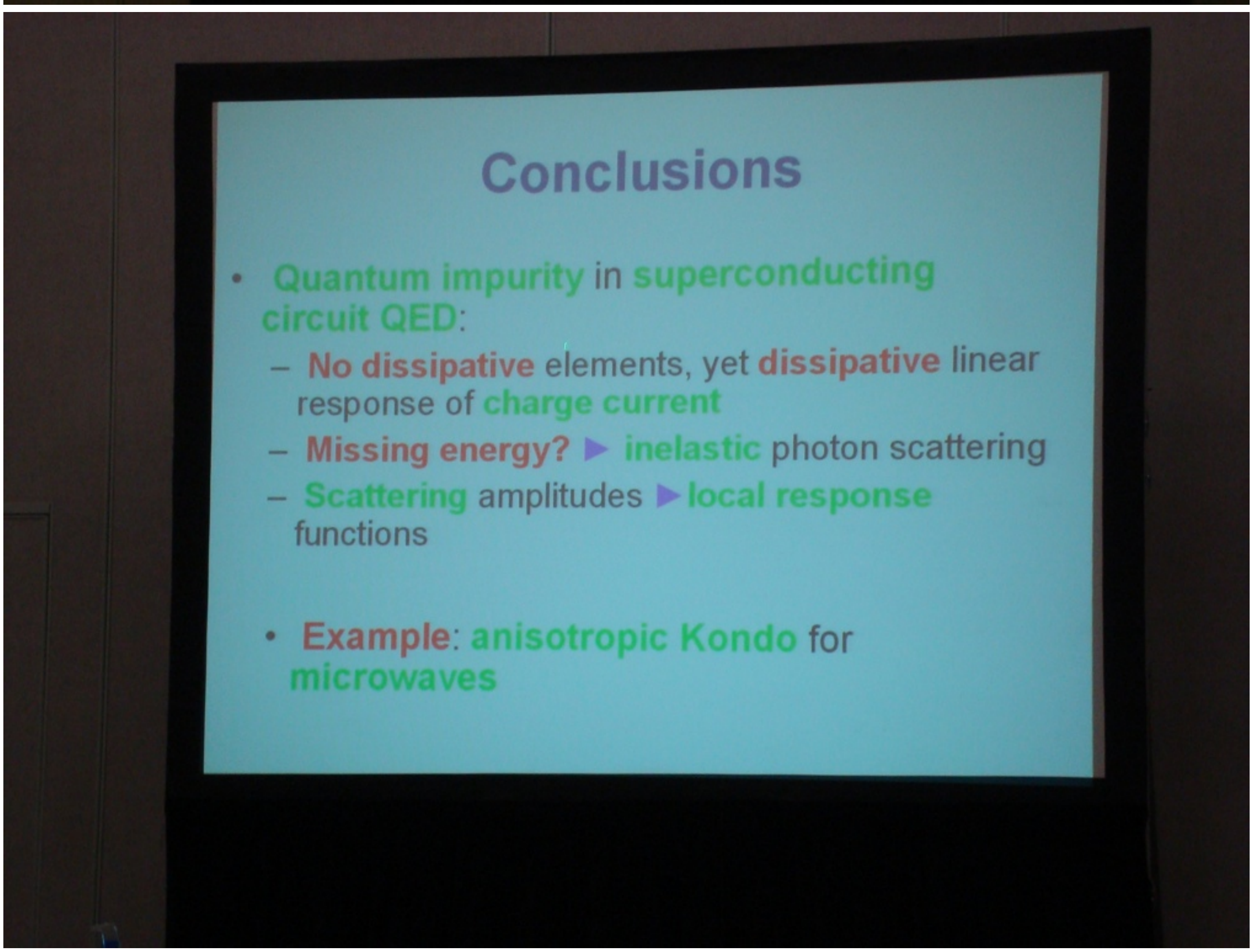

AC conductance
C'L
C'R
CLR
L
R
JL
JR
CL
CR
Iin(ω)
Iref(ω)
Itrans(ω)
• Current transmission coefficient:
$T_L(\omega) \equiv \frac{I_{\text{trans}}(\omega)}{I_{\text{in}}(\omega)} = \frac{\langle\langle i(x_{\text{in}}); i(x_{\text{out}})\rangle\rangle_\omega}{\langle\langle i(x_{\text{in}}); i(x_{\text{out}})\rangle\rangle_\omega^{(0)}}$
linear response
$i(x) \propto \partial_x \phi(x)$
($x_{in}<0$, $x_{out}>0$)
– Generalization (T>0) of photon elastic transmission amplitude


Elastic Scattering
• Elastic T matrix:
$\hat{T}^{\text{el}}_{\ell'|\ell}(\omega) = \begin{bmatrix} R_L(\omega)-1 & T_R(\omega) \\ T_L(\omega) & R_R(\omega)-1 \end{bmatrix}$
• Related to local spin susceptibility (by equations of motion):
$\hat{T}^{\text{el}}_{\ell'|\ell}(\omega) = 2\pi(-1)^{\delta_{\ell\ell'}} \alpha_\ell \alpha_{\ell'} \omega \chi_{zz}(\omega)$
$\alpha_\ell = \frac{1}{\sqrt{g}}\left(1 - \frac{CU_\ell}{a}\right)$
$|T_L(\omega)|$
T=0
0
ω
T_K
• Scattering amplitudes nonmonotonic
– reveal new low energy scale:
$T_K \sim \omega_0 \left(\nu I_{xy}\right)^{1/(\nu I_z)}$
ω_0 – bandwidth
ν – local DoS


Is Scattering Fully Elastic?
• Rate of energy loss / total inelastic scattering probability for a photon at frequency ω
$\gamma_\ell(\omega) = 1 - |T_\ell|^2 - |R_\ell|^2 =$
$4\pi\alpha_\ell^2\omega \,\mathrm{Im}[\chi_{zz}(\omega)] - 4\pi^2\alpha_\ell^2(\alpha_L^2+\alpha_R^2)\omega^2|\chi_{zz}(\omega)|^2$
• Zero for linear systems
– Kondo at $\omega \ll T_K$: vanishes to $O(\omega^2)$ [Nozieres]
• Nonzero in general !
– Kondo at $\omega \gg T_K$: $O([\nu I_{xy}/2]^2)$ – dominating over elastic transmission, $O([\nu I_{xy}/2]^4)$


Inelastic Spectrum (I)
• # of outgoing photons in lead ℓ' at frequency ω' per each incoming photon in ℓ and ω:
– linear response in energy flux– 2nd order in charge current
$\gamma_{\ell'|\ell}(\omega'|\omega) = \frac{\dot{G}^r_{n_{\omega'};i;i}(\omega,-\omega)}{\dot{G}^{r;(0)}_{n_{\omega};i;i}(\omega,-\omega)}$
• Can be reduced to local correlators (Keldysh formalism):
$\gamma_{\ell'|\ell}(\omega'|\omega) = \pi\alpha_\ell^2\alpha_{\ell'}^2\omega\omega'\left[\langle S_z^c S_z^c S_z^q S_z^q\rangle_{\omega',-\omega';\omega,-\omega} + \cot\frac{\omega'}{2T}\left(\langle S_z^c S_z^q S_z^q S_z^q\rangle_{\omega',-\omega';\omega,-\omega} - \langle S_z^q S_z^c S_z^q S_z^q\rangle_{\omega',-\omega';\omega,-\omega}\right)\right]$
2rd order response of spin correlation
3rd order susceptibility


Inelastic Spectrum (II)
• To lowest (2nd) order in I_{xy}:
$\gamma_{\ell t \ell}(\omega'|\omega) = 4\pi\alpha_\iota^2\alpha_c^2 \frac{(I_{xy}/4\pi\alpha)^2}{\omega\omega'} \times$
$\{\theta(\omega-\omega')[1+n_B(\omega')][1+n_B(\omega-\omega')] - n_B(\omega')n_B(\omega-\omega')\}\,\mathrm{Im}\langle\langle e^{i\alpha\phi(0)}; e^{-i\alpha\phi(0)}\rangle\rangle_{\omega-\omega'}$
$+ \bullet\bullet\bullet \}$
• Bolzmann equation:
ω
ω'
ω−ω'
• Leading to (T=0):
$\gamma_{\ell t \ell}(\omega'|\omega) = \pi\alpha_\iota^2\alpha_c^2(\nu I_{xy})^2 \frac{\omega_0}{\omega\omega'}\left(\frac{\omega-\omega'}{\omega_0}\right)^{1-2\nu I_z}$
0
ω'
ω


Conclusions
• Quantum impurity in superconducting circuit QED:
– No dissipative elements, yet dissipative linear response of charge current
– Missing energy? ► inelastic photon scattering
– Scattering amplitudes ► local response functions
• Example: anisotropic Kondo for microwaves