

# Probing Majorana Fermion edge states with a flux qubit

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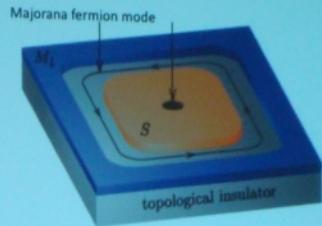
Reference: arxiv:1101.0604



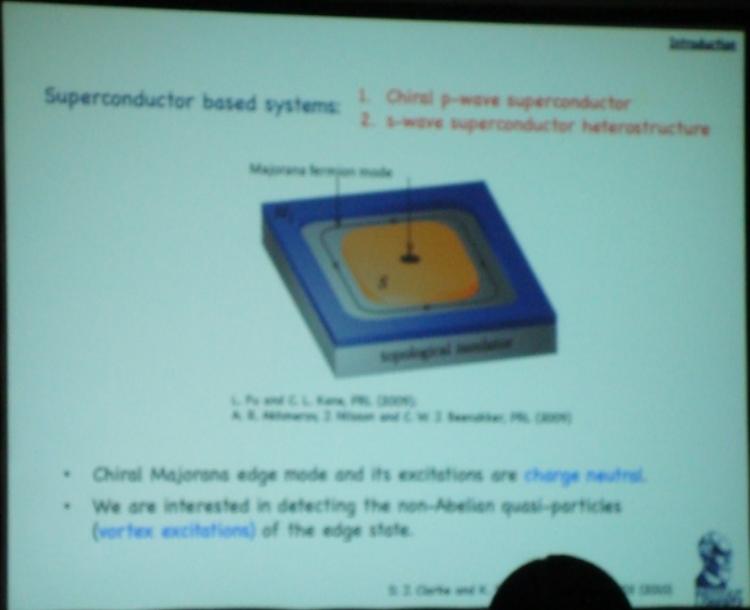
Introduction

Superconductor based systems:

1. Chiral p-wave superconductor
2. s-wave superconductor heterostructure

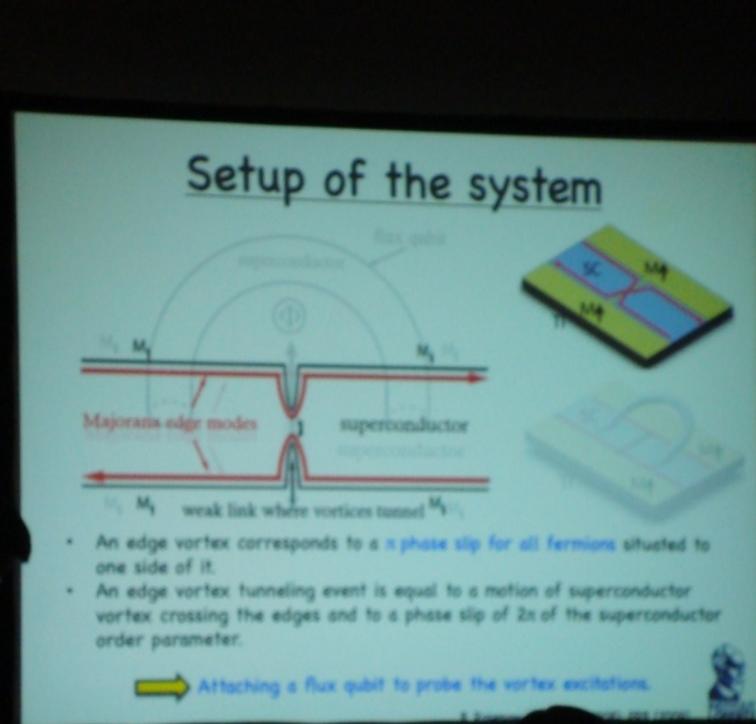
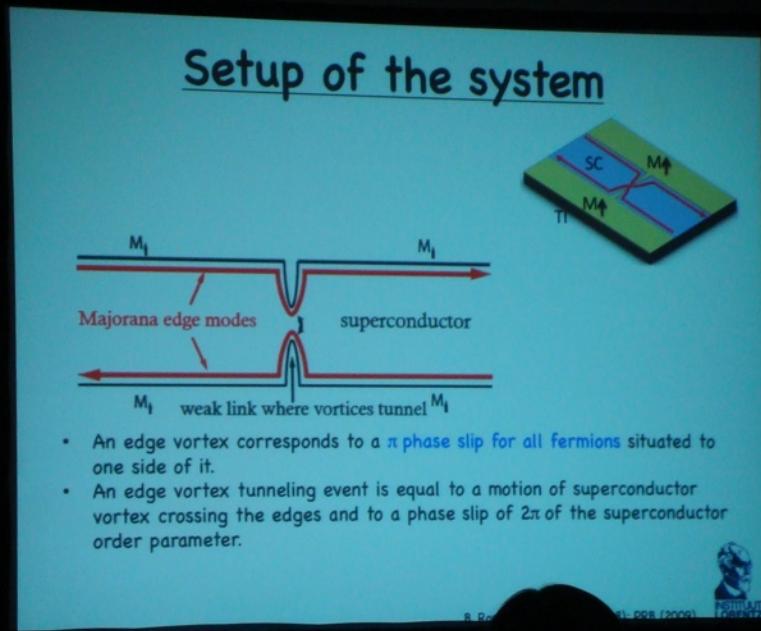


L. Fu and C. L. Kane, PRL (2009);  
A. R. Akhmerov, J. Nilsson and C. W. J. Beenakker, PRL (2009)

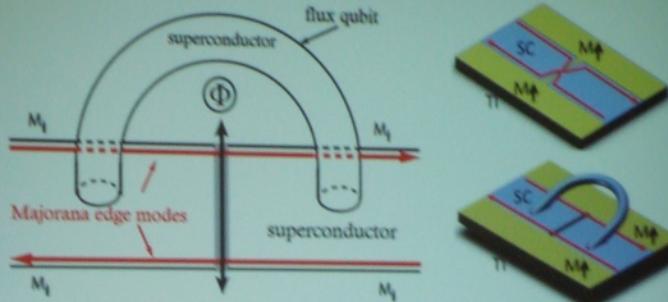


**Goal:** A way to probe the edge vortex excitations without the needs of charge degree of freedom.

Aim of the present work is to probe the vortex excitations rather than a direct application for topological quantum computing.



## Setup of the system



- An edge vortex corresponds to a  $\pi$  phase slip for all fermions situated to one side of it.
- An edge vortex tunneling event is equal to a motion of superconductor vortex crossing the edges and to a phase slip of  $2\pi$  of the superconductor order parameter.

→ Attaching a flux qubit to probe the vortex excitations.

B. Roushan et al., *Nature Physics* 5, 162 (2009)



## Majorana Fermion and Ising Model

Chiral Majorana Fermion modes:

$$\text{Superconductor} \quad H_{\text{MF}} = \frac{i v_M}{2} \int \frac{dx}{2\pi} [\psi_d(x) \partial_x \psi_d(x) - \psi_u(x) \partial_x \psi_u(x)]$$

$$H_I = -J \sum_n (s_n^x s_{n+1}^x + s_n^z)$$

Transverse field Ising Model at critical point

$$\Lambda \approx \Delta \mapsto J, \quad v_M \mapsto 2Ja \quad \text{cutoff}$$

Chiral Majorana Fermion modes with  $2\pi$  phase slip (vortex tunnelling):

$$2\pi \quad \text{Superconductor} \quad 0 \quad H_{\text{MF}} \mapsto P H_{\text{MF}} P \quad \text{with} \quad P = \exp \left[ i\pi \int_{-\infty}^{x_0} dx \rho_e(x) \right]$$

In the Language of Ising Model:  $P H_{\text{MF}} P \mapsto \mu_{n_0+1/2} H_I \mu_{n_0+1/2}$

$$P \mapsto \prod_{j \leq n_0} s_j^z \equiv \mu_{n_0+1/2}^x \quad (n_0 a = x_0) \quad \mu \text{ Disorder field}$$

→ Edge Vortex tunneling event is the discrete version of the Ising model.

## Expectation values

To the lowest non-trivial correction:

$$\langle \sigma^x \rangle = \frac{\Gamma(\frac{3}{4})\delta}{\varepsilon^{1-2\Delta_\mu} \Lambda^{2\Delta_\mu}}$$

$$\langle \sigma^y \rangle = 0$$

$$\langle \sigma^z \rangle = 1 - \frac{3\delta}{8\varepsilon} \langle \sigma^x \rangle$$

- Scaling dimension of the disorder field:  $\Delta_\mu = 1/8$
- Anomalous scaling  $\left(\frac{\varepsilon}{\Lambda}\right)^{2\Delta_\mu}$  due to the fact that each spin flip of the qubit spin couples to a disorder field  $\mu$ .

→ Measuring  $\varepsilon$  dependence of the expectation values gives the scaling dimension of the disorder field (vortex excitation).

